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THE ABILITY OF HIGH SCHOOL PUPILS
TO SELECT ESSENTIAL DATA IN
SOLVING PROBLEMS

BY
BENJAMIN W. DAILY, PH.D.

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THE ABILITY OF HIGH SCHOOL PUPILS TO SELECT ESSENTIAL DATA IN SOLVING PROBLEMS

CHAPTER I

THE SUBJECT AND METHOD OF THE STUDY

This study is an attempt to ascertain the extent to which first-year high school pupils possess and exercise the ability to select from abundant data in a school problem those which are pertinent, and to organize them so that solving will produce a correct answer; or, when essential data are lacking, to what extent pupils detect their absence and demand additional data. The scope of the problem is comprehended in the following questions:

1. To what extent are problems which contain superfluous or insufficient data recognized and used by authors of text books and by teachers of high school mathematics?

2. What is the effect of introducing some problems which contain superfluous or insufficient data among problems with which pupils are familiar?

3. To what extent can the habits of selecting and demanding essential and adequate data for the solution of problems be developed by suitable instruction?

4. To what extent are such abilities and habits as may be developed in connection with mathematical problems, transferred to similar problematic situations in other subjects?

A common criticism of education in schools is that pupils are not trained in habits of reflective thinking that are needed outside of the school room. Pupils may be at ease in situations to which they have been accustomed, but are lost when facing unusual situations where genuine thinking is necessary. Schools are held responsible for the deficiencies with the charge that pupils are taught to memorize masses of material and to follow rules mechanically, but are given little opportunity to exercise judgment. Such criticisms have suggested this study of the types of verbal problems ordinarily found in text books and courses of study.

John Dewey's definition of thinking is accepted here as pre-

senting the actual mental processes involved in skilled thinking. According to Dewey,¹ the analysis of thinking into steps or processes merely indicates the logical movements involved in a critical act of thought. It does not imply the temporal order in which the steps must occur, for in actual reasoning the steps may be interwoven and fused. Some of his fundamental ideas, for the purposes of this study, are contained in the following paragraphs from *How We Think*.²

The origin of thinking is some perplexity, confusion or doubt. Thinking is not a case of spontaneous combustion; it does not occur just on general principles. There is something specific which occasions and provokes it. General appeals to a child to think, irrespective of the existence in his own experience of some difficulty that troubles him and disturbs his equilibrium, are futile as advice to lift himself by his boot straps.

Given a difficulty, the next step is suggestion of some way out, the formation of some tentative plan or project, the entertainment of some theory which will account for the peculiarities in the question, the consideration of some solution for the problem. The data at hand cannot supply the solution; they can only suggest it.

If the suggestion that occurs is at once accepted we have uncritical thinking, the minimum of reflection. To turn the thing over in mind, to reflect, means to hunt for additional evidence, for new data, that will develop the suggestion and will either, as we say, bear it out or else make obvious its absurdity and irrelevance. Given a genuine difficulty and a reasonable amount of analogous experience to draw upon, the difference par excellence, between good and bad thinking is found at this point. The easiest way is to accept any suggestion that seems plausible and thereby brings an end to the condition of mental uneasiness. Reflective thinking is always more or less troublesome because it involves overcoming the inertia that inclines one to accept suggestions at their face value; it involves willingness to endure a condition of mental unrest. Reflective thinking, in short, means judgment suspended during inquiry, and suspense is likely to be somewhat painful.

The most important factor in the training of good mental habits consists in acquiring the attitude of suspended conclusion and in mastering the various methods of searching for new materials to corroborate or to refute the first suggestions that occur. To maintain the state of doubt and to carry on systematic and protracted inquiry—these are the essentials of thinking.

Thinking in the sense used here is essentially problem solving. A problem is a question involving doubt or difficulty. Whenever thoughtful search is made for means of dealing with the

¹ Dewey, John, in *Journal of Philosophy*, 19: 29-33 (Jan. 5, 1922).

² *Ibid.*, pp. 12-13.

doubt or difficulty there is reflective thinking. Among the actual processes involved are: (1) the ability to get a problem clearly in mind, and to keep it in mind in the midst of abundant data and tempting side-leads; (2) an attitude of caution or suspended judgment while the data are canvassed and sifted to see if all phases of the problem are being considered; (3) the ability to realize sufficiency or insufficiency of data, and unwillingness to proceed without additional facts when they are needed; (4) the ability actually to organize and use the data selected as adequate, in reaching a conclusion which satisfies the original need and therefore solves the problem.

On the basis of this understanding of thinking, we can proceed to ascertain the extent to which first-year high school pupils possess and exercise these abilities in the solution of school problems. It should be emphasized that not only the abilities to select and demand data are involved, but also the habits of doing so. Pupils may be capable of making selection and yet be unaccustomed to doing so because school problems ordinarily do not require it.

Opportunities for problem solving appear in many school studies, particularly in mathematics, the natural sciences, and all the social studies, history, civics, and literature. This study has been limited to the applied problems in beginning algebra and the studies which commonly parallel first-year algebra.

It is a general practice to include in school problems no facts that do not have to be used to obtain the answers, and to give in each problem all the facts needed to solve it. The problems which ordinarily are found in arithmetic and algebra text books are of a type which may be illustrated by this simple problem: "The distance from Chicago to New York City is 900 miles. First-class fare is charged at the rate of $3\frac{1}{2}$ cents a mile. What is the cost of a ticket from New York City to Chicago?" In such a problem all the information necessary for its solution is given, and only that information.

Problems of everyday life usually are more complicated. Frequently problems are met where insufficient information is supplied, for example, "Mr. Smith is going from New York City to Chicago on a train that carries Pullman passengers only. The distance is 900 miles and the cost of transportation alone is $3\frac{1}{2}$ cents a mile. What will be the cost of the journey?" The

answer cannot be given until the cost of the Pullman and other necessary expenses are known. Or problems are met which involve selection from a body of information, some of which is irrelevant to the solution of the immediate problem. Again, "Mr. Smith is going from New York City to Chicago, a distance of 900 miles, sleeping one night in a Pullman car, and eating the necessary meals on a dining car. Train No. 16 leaves New York City at 5 P.M., reaching Chicago at 10 A.M. next day. Train No. 20 leaves New York City at 10 A.M., reaching Chicago at 5 P.M. next day. Mr. Smith estimates that time saved between 8 A.M. and 5 P.M. is worth \$5 an hour to him. A ticket from New York City to Chicago on Train No. 20 costs \$35. On Train No. 16 a ticket costs 10 per cent more, because that train costs the railway company for operation $\frac{1}{4}$ of a cent more per mile per passenger than does Train No. 20. By which train will he travel most economically? What will be the total cost of the journey by this train?"

Problematic situations of life generally are more complicated than the examples given above. Problems in the world at large are seldom so well defined as either example, and situations generally, if not always, are complicated by many elements of little or no relevance to the correct solution. Many failures in thinking are due, also, to the absence of suitable facts. In the actual practice of reasoning the data must be surveyed to see if all the pertinent facts are present. If essential facts are missing the first task is to acquire them.

The lives of pupils are in and out of school and subsequently abound in a great variety of problems involving all phases of personal and social relationships. For some of these problems the data are definitely indicated when the problems are stated, but for others there are wide ranges of irrelevant and confusing matter from which essential considerations must be extricated.

An example of a problem in which the data are fully supplied is, "How can I calculate my income tax if I am a bachelor with no dependents, and with a salary of \$2500 as an income?" The directions given on the income tax blanks are so definite that practically all competent persons would reach the same conclusion, although some selection must be made in choosing the directions to be used.³

³ Suggested by Chester C. Parker's *Types of Elementary Teaching and Learning*, Chap. V.

On the other hand, such a problem as the following one presents a mass of data in the statement, and suggests many additional facts which must be discovered and considered before a conclusion can be reached. "Is it more economical for a man who has \$2000 in his savings account drawing 4 per cent interest, to continue to pay rent of \$40 a month, or to buy a house for \$4500 through the Building and Loan Association? He can pay for the house by paying \$1500 down, and \$1.09 per \$100 every month for eleven years on the remainder. Would it be a better proposition for him to buy through a bank, paying a certain amount of cash and securing the rest of the amount by giving a mortgage?" An inexperienced person may fail to take into account such matters as the amount of interest he might receive from his savings in eleven years, or expenses for taxes, insurance, and repairs which must be met by a property owner, in addition to the payments to the association or the bank.⁴

The question of how to spend ten days of vacation pleasantly and beneficially on the amount of money available, may be a subject of extended research and profound thinking. Many a vacation has been a disappointment because not all the pertinent facts were known and considered in planning for it. "Can I afford to own an automobile? Shall I buy a Ford or a Buick? Is it cheaper to buy cord tires at \$19.50 each, or fabric tires at \$13.75?" are problems which come tangled up with irrelevant considerations from which they must be incisively separated and comprehended before they can be solved.

Advertisements frequently are seen in newspapers and magazines of bargains in merchandise. We wonder why they can be so cheap when such articles for sale about us are so expensive. The explanation is that quotations often include only a part of the total expenses, or a minimum price "and up" is given. We must pay costs of packing, or shipping, or other items not mentioned in the advertisements.

The following practical problem calls for nicety of judgment in choosing the items which pertain to the answer of the problem. "Mr. Jones decided to combine a business trip with a vacation for himself and his wife. The firm by which Mr. Jones was employed paid necessary traveling and living expenses for Mr. Jones. Additional expenses for himself and all expenses

⁴ Adapted from Neely and Killius, *Modern Applied Arithmetic*.

for his wife were to be paid by Mr. Jones. He kept track of all expenses indiscriminately in his note book. When he returned he made out a bill covering the money due him from the firm." It is conceivable that Mr. Jones experienced some difficulty in sorting the items.

Practical problems vary in complexity from momentous decisions which require long and patient reflection down to minute issues which require only a moment's consideration to complete their solution, such as "What shall I eat for lunch today?" or "How can I get a parcel quickly to a friend in another part of the city?" Small problems which call for quick decisions are so prevalent and are so important in life that we are justified in presenting pupils with many of them in school.

We do not possess information which is the result of precise investigations showing what amounts and degrees of skill in selecting pertinent data and detecting the absence of missing facts are important in solving the problems of everyday life, but the examples given suggest the desirability of the development of skill on the part of pupils in processes which commonly are needed. If it is a universal practice to rule out of school problems all elements except those which must be used and to supply all the elements needed to solve them, then pupils may not be prepared adequately to encounter problematic situations in daily life which involve superfluous and insufficient data.

It is proposed, therefore, to discover what pupils do when some problems which contain superfluous or insufficient data are introduced among the problems to which they have been accustomed. Further, if the investigation should show that pupils do not know how to deal with these natural problems, it is proposed to ascertain first, how much instruction is required to teach them to select the necessary data or to detect the absence of data; and second, to what extent, if any, habits which may be developed in connection with mathematical problems are transferred to similar problems in other school studies.

Chapter II represents a study of the extent to which problems which contain superfluous or insufficient data are recognized and used by authors of text books and by teachers of algebra. In Chapter III are reported certain empirical studies of the effects of the introduction of such problems among those with which the pupils were familiar. Chapter IV presents a descrip-

tion of precise experiments to ascertain, (1) the extent to which the abilities and habits of selecting essential data and detecting missing data can be developed by suitable instruction, and (2) the extent to which such abilities and habits which may be developed in connection with mathematical problems are transferred to similar problematic situations in other subjects of study. The practical conclusions which may be drawn are found in the last chapter.

CHAPTER II

EXTENT OF THE USE OF PROBLEMS CONTAINING SUPERFLUOUS OR INSUFFICIENT DATA

Attention was first directed to the examination of certain texts, to discover to what extent problems containing superfluous or insufficient data are definitely recognized and included in them. The texts were selected from lists submitted independently by two well-known teachers of mathematics who are familiar with the teaching of mathematics in the high schools of the country. Each text was chosen either because it has a wide circulation or because it has been favorably reviewed. The method of examination consisted of (*a*) reviewing those sections of the text which would give the author's points of view, such as the preface and the directions to pupils for solving problems, (*b*) estimating the number of applied problems in the text by scanning all of the problems, (*c*) solving or closely inspecting a minimum of 25 per cent of the applied problems taking every fourth problem. A problem was considered an applied problem when its conditions were expressed in words rather than in numbers. The search was made for single problems in which superfluous data were included for the specific purpose of giving pupils opportunity to choose the essential data or from which data were omitted for the purpose of giving the pupil opportunity to detect their absence and to request additional data. Table I shows the result of this analysis. The numbers refer to the texts.¹

There appears to have been no deliberate attempt on the part of the authors of these text books to use single problems containing superfluous or insufficient data for the purpose of teaching pupils to exercise judgment in the choice of data. Those problems which occur may have been unintentional on the part of the authors, or may have been introduced for other purposes.

¹ The key numbers do not correspond to the order of the texts as listed. As it is our purpose to discuss the common practices among writers of texts, and not to single out any one in particular, a key is available for those who may be interested.

TABLE I

OCCURRENCE IN TEXTS OF PROBLEMS REQUIRING SELECTION OR DISCOVERY
OF ESSENTIAL DATA

Text	A*	B	C	D	E
1	433	194	--	2	--
2	676	156	--	10	--
3	466	200	1	--	1
4	551	345	--	--	--
5	410	191	--	--	--
6	594	125	--	--	--
7	319	164	--	--	--
8	549	126	--	--	--
9	454	115	--	--	--
10	206	80	--	--	--
11	430	300	--	--	--
12	517	130	--	--	2
13	205	75	--	--	5
14	375	100	--	--	--
Totals			1	12	8

A* = Approximate number of problems in book.

B = Number of problems examined.

C = Number of single problems found containing superfluous numbers.

D = Number of single problems found containing redundant information.

E = Number of single problems found containing insufficient data.

TABLE II

LIST OF TEXT BOOKS EXAMINED

Authors	Title	Year
Durell and Arnold	<i>First Book in Algebra</i>	1920
Edgerton and Carpenter	<i>First Course in Algebra</i>	1923
Evans and Marsh	<i>First Year Mathematics</i>	1916
Ford and Ammerman	<i>First Course in Algebra</i>	1923
Hamilton and Buchanan	<i>Elements of High School Mathematics</i>	1921
Hawkes, Luby, Touton	<i>First Course in Algebra</i>	1917
Rugg and Clark	<i>Fundamentals of High School Algebra</i>	1919
Schorling and Reeve	<i>General Mathematics</i>	1919
Slaughter and Lennes	<i>Elementary Algebra</i>	1915
Sykes and Comstock	<i>Beginners' Algebra</i>	1922
Swenson	<i>High School Mathematics</i>	1923
Vosburgh and Gentleman	<i>Junior High School Mathematics III</i>	1919
Wells and Hart	<i>Modern High School Algebra</i>	1923
Wentworth and Smith	<i>School Algebra</i>	1913

This judgment is supported further by assertions of the authors in personal letters, from which quotations are made later in this chapter, that it has not been their intentions to use problems of these types in the text books which were reviewed. It is the consensus of opinion among the authors who were consulted that such training either has been provided or might be provided by other devices.

The nearest approach to the special types are the "informational" problems which are found in one text. These are stated in terms of information which the author says is interesting and instructive but which is necessary for the solution of the problem. For example, "Probably the largest room in the world under one roof is the passenger concourse of the Union Station in Washington, D. C. Its perimeter is 1790 feet. One fifth of its length exceeds its width by 22 feet. Find the dimensions." In an early edition of this text informational problems were scattered through the body of the text. In the recent edition such problems are grouped into one set which is labeled and placed in the appendix of the text.

A second approach to the special types is the occasional use by two authors of masses of related data about a situation from which material must be used for several problems which follow. The pupil must constantly refer to the original data and make some selection.

Pupils also receive a degree of practice in selecting data when they are required to find in their memory or tables, reference books or observation, the facts needed; or when they are asked to decide what further facts are needed in order to solve a problem.

The following is a composite list of suggestions made by the different authors of text books.

DEVICES SUGGESTED BY AUTHORS OF TEXTS FOR CAUSING PUPILS TO EXERCISE
JUDGMENT IN SELECTION OR DISCOVERY OF DATA

1. Setting forth certain data followed by a series of problems each of which uses only a few of the data.
2. Liberal information in a problem which cannot be called superfluous.
3. Use of graphs which require selection of data from a mass of data.
4. Problems requiring the pupil to see and act on the need of recalling for use in one problem, the results obtained in a preceding group.
5. Problems in a set which do not conform to the type. The pupil must determine if the material is adequate.

6. Problems in which data are given from which pupils must make their own problems, or may make several problems.
7. Pupils make problems for other members of the class. They must find and organize pertinent data.
8. Problems without numbers in which the pupil will exercise his powers in telling how he would solve the problem if the usual data were given.
9. Pupils are asked to compare answers and methods of solution with other members of the class.
10. Thought producing questions which require mathematical reasoning.
11. Pupils estimate the results of a problem before computing them.
12. Problems for completion.
13. The pupil selects the best answer from a number of responses that are given.
14. A type of problem in which choice is made between two possible sets of conditions, only one of which satisfies the problem.
15. Problems with impossible answers. While the mechanics of the problem are satisfactory, the answer fails to satisfy the problem, for example, dividing 25 bulbs equally between 2 boys.
16. Of several data in a problem, one may be much less accurate than the rest. Since the accuracy of a result is no greater than the accuracy of its least accurate datum, answers vary in reliability.
17. Problems having two answers from which choice must be made; for example, both answers are alike, apparently different; meaningless or absurd answers; interpretation of negative answers.
18. Problems which are interesting as puzzles, but have real advantage also for the mathematical student, not the least of these advantages being the practice they furnish in picking out separately the material facts in a narrative paragraph; for example, the so-called digit problems.

OPINIONS EXPRESSED BY AUTHORS OF TEXTS IN MATHEMATICS

The attitudes of the authors of the text books toward problems which contain superfluous or insufficient data are revealed by their replies to a personal letter addressed to them on the subject. Seventeen replies were received.

Fourteen of these authors attach considerable importance to the matter of training pupils to exercise judgment in the selection and discovery of data for problems. Six of them express themselves in superlative terms, such as, "Too much cannot be said for the importance of training pupils to recognize and select pertinent and extraneous data in connection with any problem." Three out of the seventeen attach no importance to such training in algebra classes.

The authors are divided in their opinions of the advisability of including such problems in text books. (1) Five are positive in their convictions that such problems should be inserted in

texts. "You are hitting on a very important matter in connection with text books in mathematics. Every list of examples should contain some examples which do not conform to type." (2) Two would use the problems under restrictions. "Problems of this kind are being inserted in our books (arithmetic) but they are not sprung on the pupil without his knowledge. Pages of examples are included which are headed "Examples Containing Unnecessary Numbers." A model solution is given, and the pupil knows definitely that every example that follows contains one or more superfluous numbers." (3) Two authors would provide for such problems by means of supplementary pamphlets to be used at the option of teachers, "for such situations would be troublesome to many teachers." (4) Four believe "it is easier for a live teacher to give the experience in selecting data than it is to develop it in texts." (5) Five are opposed to the use of such problems.

The following extracts from letters illustrate the various points of view.

Difficult to accomplish in texts:

(1) "Theoretically I think it is a very good thing to do and frequently I do so in my own teaching. I have found in the preparation of text books it is very difficult, if not impossible, to accomplish the pedagogical result which you feel to be desirable, by including very much in the way of a joker in mathematical problems."

(2) "In my judgment the pupil's inability to apply his arithmetic to life problems after leaving school is due in the main to lack of training to select the facts needed for the solution. It is easier for a live teacher to give this experience than it is to develop it in text books."

Would prove troublesome to teachers:

(1) "In the text books on which I have collaborated no serious attempt has been made to present material which contains superfluous data or data in which elements are lacking. It is my thought that this material will not shortly find a place in text books planned for wide circulation, for such situations would prove troublesome to many teachers. I do think, however, that it may not be long before supplementary pamphlets are issued which contain problem material of this sort. Such pamphlets could then be used at the discretion of teachers."

(2) "I have not found it practicable to introduce problems containing superfluous data or lacking essential elements in text books of which I have been guilty. Pupils at the stage at which we receive them have all the difficulty it is desirable for them to have in dealing with problems in which the data are sufficient and not redundant. The results of the occa-

sional presentation of such problems always has been extreme puzzlement, confusion, and exasperation. This may be due to the fact that pupils were not prepared to meet such situations. If the school would undertake the preparation it would require more time to cover the same topics which we now cover. For myself, I think I should prefer to have somebody else undertake it."

Would be unfair to pupils:

(1) "When a pupil is trying to solve a problem, he has not only the wish to succeed but the hope of succeeding, and his hope rests on his belief that his beneficent teacher has made sure that success is obtainable; he will try for an unbelievably long time and will refuse to quit his task. If we should remove the faith he now possesses in the possibility of success we should remove at the same time, the incentive to this valuable perseverance. The pupil's knowledge is not, at that stage of his career, broad enough to enable him to decide that success is impossible, although it is true that some problems could be found as to which he could decide whether success was possible."

(2) "I do not mind the omission of such problems as you indicate, particularly since, so far as my experience goes, ordinary pupils have plenty to do if they master the 'first approximation' style of problem; that is, the ordinary kind wherein there is no ulterior question concerning the validity of data, or other outside matters."

Would necessitate neglect of fundamental principles:

(1) "I recognize the value of requiring a pupil to select the necessary data for his problems, but I am not in favor of doing much of it in high schools, and have not planned for it in my texts. [The expansion of the high school curriculum has produced such multiplicity of subjects that it is difficult, if not impossible, to give pupils grasp of fundamental principles and skill in applying them.]² The ability of the average high school pupil is lower than it was a few years ago. . . . This forces us to adapt the work to a new type of pupil and as a consequence, we do not get as high grade results as in former years. I believe it is more helpful to the pupil to be held to the few things we are giving him and to have him repeat them as often as possible, than to require him to scatter his energy."

(2) "We have paid little attention to the matter of problems containing superfluous data in our algebra. We are kept busy in preparing boys for the New York Regents and the College Board Examinations, and have to follow the specifications very closely. I believe this is an important topic, and in the revision of our arithmetic certain pages of examples of this kind are inserted."

The problems are not practical and are unnatural:

(1) "It was our feeling in the preparation of the text that such problems had no place in a high school text book. They can hardly be con-

² Paraphrase of the original quotation.

sidered practical problems for this reason. Problems which arise from collected material are not submitted to engineers or others until all the data are present. The scrutiny of a problem for sufficient data comes previous to a call for its solution, and in general the required data are known and the particular working out of the problem is to secure it. It seems to me unnatural to present a problem which nominally calls for a solution in which data are lacking. If problems of such a type are presented at all I feel that they should be presented merely for analysis and not for solution."

(2) "With respect to the particular kind of problem you suggest, I have been somewhat skeptical. Teachers have generally found fault with these problems because they give the children the idea that they are being unfairly treated. As a matter of fact these problems do not seem to represent the conditions which people ordinarily meet when they use arithmetic. (Example omitted.) I think this is the reason why teachers have not looked upon such problems as fair."

Valuable space is saved by omitting such problems:

"There are two reasons why we give enough and neither more nor less information in the statements of our problems. First, pupils and teachers would find such statements a source of confusion in an already difficult part of their work. Second, space in the text is at a premium. Space could be gained by giving insufficient data, and possibly that would be an advantage, for then there would be space for redundant data."

The omission is a mistake:

(1) "You are hitting upon a very important matter in connection with text books in mathematics. I have insisted that in connection with every set of examples there should be some which do not conform to the type."

(2) "Your letter touches upon a point in which I am so much interested that I am in danger of making this letter too long for you to read. (In making our texts we) had the question you raise distinctly in mind, and attempted to meet it throughout the problem material of the books."

(3) "Our most definite attempt to strike at what you have in mind is found in a new text which will be off the press soon. . . . In that book you will find a chapter on problem solving. The topics include: Problems Buried in Non-Essentials, and Problems with Missing Data."

ATTITUDES OF HIGH SCHOOL TEACHERS

Practices among teachers and their attitudes toward the use of the special types of problems, are revealed by the replies of 103 classroom teachers to a questionnaire, inquiring concerning (1) the texts used, (2) the occurrence of problems which contain superfluous or insufficient data, (3) the importance which teachers attach to the use of such problems, (4) other means em-

ployed to teach pupils to reject irrelevant material or to demand additional data when they are missing.

The questionnaire was sent to 130 teachers of algebra in the first class high schools of the country. Names were chosen from the mailing list of the *Mathematics Teacher*, published by the National Council of Teachers of Mathematics, taking eight names from each page of the list. Replies were received from 103 teachers distributed over forty states. Delaware, Florida, Kentucky, Mississippi, Nevada, Rhode Island, West Virginia, and Vermont are not represented. The high percentage of replies may be attributed in part to a follow-up letter to teachers who did not reply to the first request.

Two of the texts previously analyzed are used by fifty per cent of the schools reporting. Each of the other texts of that list is used by from one to seven of the schools reporting. Each of the following additional texts was reported by one or two schools: Breslich, *First Year Mathematics*; Milne, *Standard Algebra*; Hopkins and Underwood, *Elementary Algebra*; Reitz, Crathorne, Taylor, *First Course in Algebra*; Sommerville, *Elements of Algebra*; Stone and Millis, *Elementary Algebra*; Schultze, *Elements of Algebra*.

Of 103 teachers who replied to the questionnaire, 82 find no problems which they recognize as containing superfluous or insufficient data, 14 find such problems, and 7 do not reply to the question.

While the teachers find few such problems in the texts they are using, 58.5 per cent of the total number of teachers to whom questionnaires were addressed expressed a sentiment favoring the use of problems which will cause pupils to exercise judgment in choice of data; for example, 44 teachers attach much importance to such problems, 32 attach some importance. On the other hand, 41.5 per cent may be counted as not favoring their use; e. g., 6 teachers say they see no value in the problems, 21 do not answer the question, and 27 did not reply to the questionnaire.

The following extracts from remarks by teachers illustrate the attitudes and practices among them.

Objections on the grounds that teaching would be made a more difficult task than it is:

"It seems difficult enough to teach the fundamental concepts in freshman algebra with only necessary facts presented, without interjecting

superfluous material. Pupils do well enough if they interpret that type of problems which has just the necessary data."

"Problems of the kind suggested might give valuable training, but there are so many skills which need to be developed or which the course of study calls for, that we have not time for the problem material."

"The main work of the ninth grade is to teach the use of the equation. If we teach all the required algebra and the use of such data as are given, we have little time to complicate the problems by introducing superfluous data."

"Our main problem is not that of teaching pupils what are to be selected, but what to do with them when proper data are given. If the pupils understand this they will experience little or no difficulty in selecting the proper data in a real life problem."

First-year high school pupils are too immature to solve such problems:

"The disadvantages outweigh the advantages. Most pupils in first-year algebra are too immature to recognize that material is irrelevant. It is difficult for them to get relationships between necessary elements. Such problems might be valuable later in the course."

"It would be fine training for capable pupils, but the majority cannot solve problems when all data needed and only such data, are given. Many teachers omit most of the practical problems, and at best, are not successful in teaching them."

"Such problems should not be introduced until the mathematical principles and procedure has been given and are clearly understood. After that, well selected problems of the types mentioned would be valuable."

Pupils get training in other subjects:

"Pupils get practice of this kind mostly in field work, in investigations, in current events, and debates."

"The first training that a high school pupil gets in rejecting irrelevant material or demanding additional data, is in the courses in physics and chemistry."

Use of such problems has doubtful value:

"It seems very good in theory; and I have used some such problems of my own devising to some advantage. In a text, without supervision, they might not work so well."

"I do not see how these problems could be handled successfully in texts. Is this not an argument for the project method of teaching mathematics?"

Occasional use is desirable.

"A few such problems are quite worthwhile,—anything to make the pupils think. They like to run up against a catch problem now and then."

"These problems are valuable for the purpose of arousing interest rather than of giving examples of future contingencies."

"If there were enough problems with either superfluous or insufficient data to cause the pupils to be alert to recognize either, the training would be valuable. A few widely separated problems would cause confusion."

The problems have much value:

"Men and women must be able to select and reject data. High school is a good place to learn this. Mathematics is a good medium."

"The idea is a splendid one, if problems can be stated so that they will require a bit of real thinking on the part of the pupils."

"I am much interested in this inquiry and I hope it will be a means of correcting a serious error in our text books. Our texts provide too little material to properly train pupils in discrimination."

"Personally, I believe such problems may open the way to a better grasp of the verbal problems. These problems add a desirable element in training."

"Experience in the classroom shows that pupils take great interest in these problems and that they are very profitable to them, when the pupils are aware of the nature of the problems and hence do not feel that they are being tricked. As a matter of fact this is the situation in the external world, where a pupil knows that superfluous data may be connected with his problems."

"Since life situations seldom provide just that material which is essential to the solution of the problem at hand, it is my thought that training in meeting complex problems situations of the type you describe, should command the consideration of teachers of mathematics and science in our secondary schools."

CONCLUSIONS

The conclusions may be drawn that problems which contain more data than are needed to solve them and problems some of the data for which are lacking, seldom occur in text books and are not used commonly by teachers of high school algebra. While teachers and authors occasionally make use of such problems and other means for stimulating judgment concerning data, the practice is well established of giving in a problem the exact data needed to solve it, no more and no less. Since the problems are not found in representative text books they would not commonly be used by teachers generally.

The question of the use of such problems is not a new one. When teachers and writers of texts have considered it the question has been answered generally on the conservative side. The principal objections have been: (a) The average first-year high

school pupil is not capable of exercising the necessary judgment; (b) the effect of the occurrence of such problems has been to upset and exasperate both teachers and pupils; (c) their introduction would complicate seriously the task of teaching in a phase of the work which already is difficult.

The questions raised by these objections are: (a) What are the effects upon pupils of the introduction of the special types of problems? (b) Can pupils be taught to select the necessary data and to detect their absence when they are lacking? (c) What amount of instruction is necessary to teach pupils to solve such problems successfully? If precise experiments reveal that the special types of problems contribute to the development of abilities and habits which are involved in skillful thinking in and out of school, then the question of the introduction of such problems into courses of study and text books may be considered.

The following chapters describe studies and experiments which were made in an attempt to obtain objective answers to these questions.

CHAPTER III

EFFECT OF INTRODUCTION OF PROBLEMS WHICH CONTAIN SUPERFLUOUS OR INSUFFICIENT DATA AMONG PROBLEMS WITH WHICH CHILDREN ARE FAMILIAR

Problems with which the children were familiar were found and superfluous data were added to these to form the preliminary tests. Based on these data the following studies in Chapter III were made:

- A. Formulation of preliminary sets of test problems.
- B. Description of the preliminary studies.
- C. Character of the pupils' responses, shown by (1) classification of 1000 responses; (2) illustrations of different types of answers.
- D. Comparison of success of pupils on special problems with success on corresponding regular problems.
- E. Extent to which the results are general among high school pupils.

A. FORMULATION OF PRELIMINARY SETS OF TEST PROBLEMS

As problems of the types which it was desired to use in the investigation occur rarely in text books which are in common use, it was necessary to devise such problems. Many problems found in algebra texts have to do with unnatural subjects and unreal practices. While no attempt is made to justify teaching these problems, this study has been confined to the types of applied problems which occur commonly in texts, and to which pupils are accustomed. In formulating problems containing superfluous data, care was taken that the redundant data should be pertinent to the situation and not merely complicating or confusing.

An analysis was made of the applied problems in the texts previously reported, and a list was made of types common to

fourteen texts. These were classified as to type of subject matter and mathematical computations involved, the relative frequency of each type was estimated, and the phraseology most commonly found was utilized. This list of problems was submitted to the criticism of a group of three high school teachers of algebra, three college teachers, and three graduate students in education. On the basis of their judgments selection was made of the problems with which high school pupils in general probably would be familiar.

Problems from the list thus derived were modified by inserting superfluous data and subtracting essential elements, and new problems were formulated which were similar to old ones except in the experimental elements. These problems also were submitted to the criticism of the same judges, and selection was made of those which were most representative of the desired factors. The final selection consisted of sixteen problems classified under four heads: (1) Four problems containing superfluous numbers. (2) Four problems containing redundant information. (3) Four problems in which the data are insufficient to obtain an actual answer, but in which the results may be represented by algebraic expressions. (4) Four problems containing insufficient data, but which the ordinary pupil in first-year algebra would not be prepared to represent by algebraic expressions.

Crude Standardization of the Tests. The special problems were included among sets of corresponding regular problems. Objective information to aid in perfecting these tests in subject matter and phraseology was obtained by trying out the problems in four schools in and near New York City. The number of problems which pupils would solve in a forty-minute period and the degree of difficulty permissible, were determined.

Resulting Eight Sets of Preliminary Text Problems. Seven sets of problems were perfected to be used in the preliminary studies, and from the results revealed by them, an eighth set was derived.

Sets I to VI consist of six problems each, four of which are of the types to which pupils ordinarily are accustomed and two are of the special types. Each special problem is paralleled by a regular problem which corresponds to it in the essential mathematical reasoning and computations which are involved. In this report the contrasted problems will be referred to as Special

problems and Corresponding Regular problems. Thirty-five minutes were found to be sufficient for all except very slow pupils to finish a test.

Set VII consists of ten problems, four of which are special and six are regular. Each special is paralleled by a corresponding regular. Since the test is too long for one class period it is divided into two parts which were administered on successive days.

Set VIII is made up of the twelve problems which gave the most satisfactory results in the earlier sets. It consists of two parts, each of which contain three special and three corresponding regular problems.

B. DESCRIPTION OF THE PRELIMINARY STUDIES

The purposes of the early studies were (a) to find the types of responses which pupils make to the special problems; (b) to obtain an indication of the relative success of pupils on the regular and on the special problems; (c) and to discover whether the results are general among first-year algebra pupils. The tentative conclusions from these studies were later tested in controlled experiments which are reported in Chapter IV. The first seven sets of problems were given each to a minimum of one hundred pupils divided among at least three classes in widely distributed schools,¹ under varying conditions of learning and teaching. The status of each class, in respect to the text used and the sections in which the class had been instructed, had been ascertained by correspondence with the principal of the school and the teacher of the class. All pupils were in their second half-year of algebra. Classes were reported by their teachers to be average in ability, except two classes which were reported superior in ability. Sets of problems were used which seemed to be adapted to the experience of the class.

C. CHARACTER OF PUPILS' RESPONSES TO SPECIAL PROBLEMS

It is desirable to analyze what pupils do when confronted by new and perplexing situations. Do they do anything? Are they confused by the novel problem? Are they misled by extraneous suggestions? Do they indicate that they are accustomed to meeting such problems? Do they make successful adjustment?

¹ The schools are listed in Table IV, page 44. A total of 1056 pupils were tested in 36 classes from 16 schools in 9 states.

Do they exercise judgment in selecting or demanding data? Are they interested? pleased or displeased? Are they discouraged?

Means of Ascertaining Pupils' Responses. Data upon the pupils' reactions were secured by the form of the test and by the specific directions to the pupils. A practice problem was followed by: "On the following pages are problems similar to this one. In the space below each problem write out a complete solution. Do all your work on this paper. Do not spend so much time on one problem that you do not have time to try them all. *If for any reason you do not solve a problem tell why you do not. You may write on this paper anything you think about a problem but do not ask any questions aloud. You must decide for yourself what to do.*" These directions stimulated a variety of responses and freedom of expression by the pupils.

In order to make test conditions as nearly uniform as possible, specific and detailed directions were given to teachers administering the tests, and they were asked for written reports relating to test conditions and giving their judgments of the pupils' responses.

TABLE III

CLASSIFICATION OF 1000 RESPONSES TO THE SPECIAL PROBLEMS, BY FOUR HUNDRED AND FIFTY PUPILS FROM TWELVE SCHOOLS

Response	Per Cent	Per Cent
1. Completely baffled	11.5	--
a. No answer	--	8.3
b. Incorrect answer, equation or solution omitted	--	0.5
c. Incomplete or inappropriate solution	--	2.7
2. Confused	21.1	--
a. An inadequate or irrelevant excuse for not solving a problem	--	15.0
b. Apparent disregard of superfluous data, but failure to comprehend the problem	--	5.6
c. Mistaken judgment concerning adequacy of data	--	0.5
3. Undismayed but in error	35.2	--
a. Attempted solution of a problem in spite of absence of essential data	--	25.0
b. Attempted solution by use of superfluous data	--	10.2
4. Correct reasoning with erroneous computations	4.9	--
a. With disregard of superfluous data	--	3.3
b. With detection of absence of data	--	1.6
5. Completely successful	27.3	--
a. Disregard of superfluous data, with correct solution	--	16.3
b. Absence of essential data given as reason for no solution, or request for more data	--	10.3
c. An algebraic expression, offered as answer to an insufficient data problem	--	0.8

Scoring the Tests. The following standards were adopted for judging pupils' success on the different types of problems.

Ability to solve a problem shall be understood to involve ability to read and understand the text, to select the essential data, to formulate an equation which represents the condition of the problem, and finally to solve the equation. The following questions are to be answered yes or no:

1. In a regular problem, (a) does the pupil formulate an equation which represents the conditions of the problem?
(b) Does he successfully complete the solution?
2. In a superfluous data problem, (a) does the pupil disregard superfluous and select essential data? (b) Does he use the data selected in reaching a correct solution?
3. In an insufficient data problem, does the pupil detect the absence of data, and (a) refuse to solve the problem or (b) demand additional data?

Solutions were classified as unsuccessful, partially successful, and fully successful. A solution was called partially successful when the pupil gave evidence of having interpreted the problem correctly but failed on the solution. Correct interpretation together with correct solution was called fully successful. Failure in both interpretation and solution was called an unsuccessful solution.

The detailed instructions for scoring were as follows:

UNSUCCESSFUL RESPONSES—SCORE 0

No answer.

A wrong answer, the equation or solution omitted.

Incorrect or incomplete solution.

Disregard of superfluous data in a problem, but lack of comprehension of the problem.

An irrelevant or inadequate reason for not solving a special problem.

Attempted solution of a problem in spite of the absence of essential data.

Attempted solutions by making use of superfluous data.

Mistakes in judgment concerning adequacy of data.

PARTIALLY SUCCESSFUL RESPONSES—SCORE 1

Correct equation or method, mistake in computations, combined with disregard of superfluous data.

Correct equation or method, mistake in computations, combined with detection of absence of essential data.

Correct answer, equation or solution being omitted.

FULLY SUCCESSFUL RESPONSES—SCORE 2

Correct equation and computations, combined with disregard of superfluous data.

[Continued on next page]

Correct equation and computations, combined with detection of absence of essential data.

Conditions of an insufficient data problem represented by an algebraic expression, which is offered as an answer.

Illustrations of Different Types of Answers. Certain typical answers are given below. They are listed as to (1) superfluous numbers, (2) superfluous information, (3) insufficient data, (4) both superfluous and insufficient data.

(1) SUPERFLUOUS NUMBERS

Problem: The capacity of a certain swimming pool is 133,000 gallons. It can be filled in 40 hours by one pipe which has 5 outlets. It can be filled in 60 hours by a second pipe which has only 3 outlets. It is desired to fill the pool as quickly as possible. How long should be required if both pipes are running at the same time?

Classified Answer

1. Excuses for not solving

- a. I don't understand it.
- b. It can't be done.
- c. We did not do such problems.
- d. It is too complicated.
- e. Too many words. I get all mixed up and can not make heads or tails of it.
- f. I think there is not enough material to work with.
- g. Did not pay attention when the teacher was explaining that kind of problem.

2. Solutions Using Superfluous Data

a. Error in statement of the equation.

Juggles the results to get a reasonable answer.

8 outlets = 28 hours it will take.

x = time required.

$\frac{1}{5}x$ = outlets of first pipe in 8 hours.

$\frac{1}{3}x$ = outlets of second pipe in 20 hours.

$\frac{1}{5}x + \frac{1}{3}x = 28$ hours.

$8x = 3 - \frac{1}{2}$

b. Uses some superfluous data and rejects other. Tries multiplication.

First pipe

$$\begin{array}{r} 5 \\ \times 3 \\ \hline 15 \text{ outlets} \end{array}$$

$$\begin{array}{r} 3 \\ \times 40 \\ \hline 120 \text{ hours} \end{array}$$

$$1/120 + 1/300 = 1/x$$

$$5x + 2x = 600$$

"The amount of gallons is not needed."

Second pipe

$$\begin{array}{r} 3 \\ \times 5 \\ \hline 15 \text{ outlets} \end{array}$$

$$\begin{array}{r} 5 \\ \times 60 \\ \hline 300 \text{ hours} \end{array}$$

$$x = 85\frac{5}{7} \text{ Ans.}$$

- c. Feels that all numbers must be used, regardless of the unreasonable equation and answers.

$$\begin{array}{rcl}
 5/x + 3/y & = & 133,000 \\
 8/x + 20/y & = & 100 \\
 \\
 40/x + 24/y & = & 1,064,000 \\
 40/x + 100/y & = & 500 \\
 \hline
 76/y & = & 133,000
 \end{array}
 \qquad
 \begin{array}{rcl}
 76\ y & = & 133,000 \\
 \\
 y & = & 1,750 \\
 x & = & 13,125 \\
 y & = & 29\ 1/6\ \text{hours.} \quad \text{Ans.}
 \end{array}$$

- d. After futile attempts to use data, gives up.

$$\begin{array}{rcl}
 133,000 & & \\
 1800 & 180 & | \ 1800 \qquad 40 \qquad 3 \\
 \hline
 239,400,000 & & \qquad 5 \qquad 60 \\
 & & \hline
 & & 200 \qquad 180
 \end{array}$$

$1/200 + 1/180 = 133,000$

"I don't understand the outlets."

3. Success. Disregards all superfluous data and solves problem correctly

Let x = what both do together.

$$1/40 + 1/60 = 1/x$$

$$60x + 40x = 2400$$

$$100x = 2400$$

$$x = 24\ \text{hours, it will take.}$$

"I think you do not have to put how many gallons in the tank.
Just say a tank."

(2) SUPERFLUOUS INFORMATION

Problem A: The first appropriation for a library for Congress was made in 1800. The present Library of Congress was completed in 1897 at a total cost of six million dollars, the gold leaf on the dome alone costing \$3800. If the first appropriation had been \$1200 less than it was, it would have paid only for the gold leaf on the present library. What must have been the amount of the first appropriation?

Problem B: The pupils in a history class of fifty members have each agreed to buy a supplementary book. A book company is advertising a new book which has been adopted by many schools and is well liked. The selling price is \$2. It is printed on good paper and contains three hundred pages of reading matter. The principal can secure the books for $4/5$ of the selling price if he buys a large number of them. How much will each pupil have to pay for his book?

Classified Answers

1. Inability to isolate the question and to select the essential data
 - a. "The question is told in too complicating a way." (A)
 - b. "Too much told. Can't seem to make sense of it." (A)

- c. "Can't get an equation or anything to work on." (A)
 d. "There is nothing with which an equation can be formed." (A)
 e. "Don't know how to begin. Don't know whether to put x or \$2 for the selling price." (B)
 f. "It was not understandable." (A)
 g. "Can't figure it out." (B)
2. Resentment
 a. "Silly question." (Accompanied by erroneous solution.) (A)
 b. "Can't be done. The question is foolish." (A)
 c. "Never had this problem or anything like it." (B)
3. Caution
 a. "We don't know if the pupils got them at cost. Don't know the selling price." (B)
 b. "Impossible, because the problem is not exact and you don't know how many he gets, and 300 pages hasn't got anything to do with it." (B)
 c. "There are not enough facts given." (B)
4. Use of unnecessary data in solution (B)
- | | |
|--|---|
| <p>a. <i>versus</i></p> <p>Let x = amount each paid</p> <p style="margin-left: 40px;">$50 \times \\$2 = \\100</p> <p style="margin-left: 40px;">$\frac{1}{2}\%$ of \$100 = \$ 80</p> <p style="margin-left: 40px;">$50x = 80$</p> <p style="margin-left: 40px;">$x = 1.60$</p> | <p>b.</p> <p>Let x = amount each paid</p> <p style="margin-left: 40px;">\$2.00 = selling price</p> <p style="margin-left: 40px;">$x = \frac{1}{2}\%$ of \$2.00 = \$1.60</p> |
|--|---|
5. Success, after trial and error
 a. "Easy, but took some time to get the idea of the problem." (A)

(3) INSUFFICIENT DATA

Problem: A bus can carry 10 more passengers than an auto. To accommodate all the passengers arriving on a train, 7 autos and 2 buses will be needed. How many passengers are arriving?

Classified Answers

1. Verbal Comments
- a. "Haven't had this kind of a problem."
 b. "I cannot do it on account of I don't know whether to solve for x or for x and y ."
 c. "Doesn't seem to work. I think it is an impossible problem."
 d. "I think that it cannot be worked. Comes out 9ths and no persons are 9ths."
 e. "This cannot be done because we must know either how many people the bus or auto holds."

f. "Unable to do. Since there are 2 unknowns there has to be two equations and there has to be one more statement to form a simultaneous linear equation."

g. "I think the problems are very easy and that they ought to be able to be worked by any boy who is taking second term algebra."

2. Attempted solutions in spite of insufficient data

a. Let x = what the auto carries.

$$\begin{array}{rcl} x + 10 & = & \text{what the bus carries.} \\ \hline 2x & = & 10 \\ x & = & 5 \\ x + 10 & = & 15 \end{array} \qquad \begin{array}{rcl} 7 \times 5 & = & 35 \\ & & 30 \\ \hline & & 65 \text{ passengers} \end{array}$$

b. x = passengers carried by auto.

$$\begin{array}{rcl} x + 10 & = & \text{passengers carried by bus.} \\ 7x & & 7 + 20 + 7 = 34 \text{ passengers} \\ x + 2 & & 24 \times 2 = 48 + 70 = 118 \text{ Ans.} \end{array}$$

c. Let x = number in bus.

$$\begin{array}{rcl} x + 10 & = & \text{number in auto.} \\ 7x & = & 2(x + 10) \\ 7x & = & 2x + 20 \qquad 28 \\ 7x - 2x & = & 20 \qquad 28 \\ x & = & 4 \qquad 56 \text{ Ans.} \end{array}$$

3. Represents the conditions of the problem by an algebraic expression and offers it as an answer to the problem.

$$\begin{array}{lcl} \text{Let } x & = & \text{passengers of an auto.} \\ x + 10 & = & \text{passengers of a bus.} \\ 7x & = & \text{auto.} \\ 2(x + 10) & = & \text{bus.} \\ 7x + 2x + 20 & = & \text{passengers.} \qquad 9x + 20 \text{ Ans.} \end{array}$$

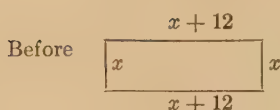
(4) BOTH SUPERFLUOUS AND INSUFFICIENT DATA

Problem A: A man had plans for a house of which the foundation was to be 12 feet longer than it was wide. Finding that it would cost \$18,000, which was more than he could afford, he decided to cut off 3 feet from the length and 2 feet from the width. In this way he lowered the cost to \$15,000. What were the dimensions of the foundation before and after making the changes?

Classified Answers

1. Solutions of the problem in spite of the insufficient data, by using the superfluous data.

a. Substitutes cost for perimeter. Is undisturbed by the unreasonable dimensions of the house.

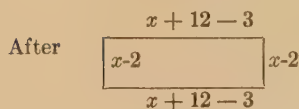


$$4x + 24 = 18000$$

$$4x = 17976$$

$$x = 4494$$

$$x + 12 = 4506$$



$$4x - 4 - 6 + 24 = 18000 - 3000$$

$$4x = 15010$$

$$x = 3752\frac{1}{2}$$

$$x + 9 = 3761\frac{1}{2}$$

Answer

Before

$$4744$$

$$4746$$

After

$$3752\frac{1}{2}$$

$$3761\frac{1}{2}$$

b. Sees apparent analogy between the problem and second degree equations with which she has had experience.

Let x = width before

$$(12 + x - 3)(x - 2) = \$7500$$

$12 + x$ = length before

$$11x + x^2 + 18 = 7500$$

$$x(12 + x) = \$9000$$

$$x^2 + 11x + 18 = 7500$$

$$12x + x^2 = 9000$$

$$(x + 9)(x + 2) = 7500$$

Let $12 + x + 3$ = length after

$$x + 9 = 7500$$

$x - 2$ = width after

$$x + 2 = 7500$$

$$x = 7491$$

$$x = 7498$$

c. An ingenious solution but in error

$$x = w$$

$$x + 12 = 1$$

$$x(x + 12) = \$18000$$

$$(x - 2)(x + 9) = \$15000$$

$$x(x + 12) - (x - 2)(x + 9) = 18000 - 15000$$

$$x^2 + 12x - x^2 - 7x + 18 = 3000$$

$$5x = 2982$$

$$x = 596\frac{2}{5}$$

Answers

$$x = 596\frac{2}{5}$$

$$x + 12 = 608\frac{2}{5}$$

$$x - 2 = 594\frac{2}{5}$$

$$x + 9 = 605\frac{2}{5}$$

2. Doubtful reasons for not solving

a. "It is too difficult."

b. "I don't understand how to go about it."

3. Recognize the peculiar features of the problem

a. "If the one that asked the problem wished to confuse the pupils it

seems they have in some way succeeded. The very length of it is a sort of puzzle and the unnecessary excess information also succeeds to entangle the pupil."

- b. After trying various solutions one pupil comes to the following conclusion: "Too much useless material and not enough useful."
- c. "I do not see what the cost of the house has to do with finding the dimensions."
- d. "Cannot be answered because feet don't give dollars. Dollars per foot is missing."
- e. Represent the dimensions of the house by algebraic expressions and give them as answers to the problem.

Problem B: The main waiting room in the Union Railway Station at Washington, D. C., has an area of 28,600 square feet. The length exceeds the width by 90 feet. How many people can be accommodated in this waiting room?

Classified Answers

1. Attempted solutions in spite of insufficiency of data, by using such data as are given
 - a. Response to the analogy of this problem to problems which involve finding dimensions when the perimeter is given.

$$w = x \quad \begin{array}{|c|} \hline 28,600 \\ \hline \end{array}$$

$$\begin{aligned} \text{Waiting room} &= 28,600 \\ 2x + 2x + 180 &= 28,600 \\ 4x + 180 &= 28,600 \\ 4x &= 28,420 \\ x &= 7,105 \text{ people.} \end{aligned}$$

- b. Juggles figures until an equation and an answer are found which appear sensible.

$$\begin{aligned} x &= \text{area} \\ x - 90 &= \text{length} \\ x + x - 90 &= \\ x + x &= 90 \\ 2x &= 90 \\ x &= 45 \\ 28,600 \div 45 &= 6355 \text{ people} \end{aligned}$$

- c. A baffled attempt.

$$\begin{array}{|c|} \hline x + 90 \\ \hline 28,600 \\ \hline \end{array} x$$

$$\begin{aligned} x^2 + 90x &= 28,600 \\ (x + \quad) (x + \quad) &= \quad 0 \end{aligned}$$

- d. Realizes a difficulty but cannot locate it.

$$x \quad \begin{array}{|c|} \hline 28,600 \\ \hline \end{array}$$

$$\begin{aligned} x(x + 90) &= 28,600 \\ x^2 + 90x &= 28,600 \end{aligned}$$

"I do not know how to finish."

2. Reasons for not solving the problem

- a. "Can't do it because I do not know the meaning of 'exceeds'."

- b. "Could not do it because could not understand the problem."
 - c. "How do you think I know? I've never seen the place or heard of it. As many as it will hold." (Resentment.)
 - d. "Can't be solved because the value of x can't be found and you can't tell how many people anyway if you only have area."
 - e. "Cannot do it because I do not know what space a person takes up and there is no way of finding out."
3. Supplies the missing data
Juggles the figures a while, then abandons futile attempts to use superfluous data and supplies the missing data, solving the problem thus: "Allowing 2 square feet per person, $28,600 \div 2$ equals 14,300."

Summary of Conclusions. That these pupils generally are not accustomed to problems which contain superfluous or insufficient data is evident from the various responses made to the test problems. Two thirds of these pupils give replies which indicate that they are wholly unaccustomed to such problems. Practically one third are confused or completely baffled by the novel problems. Fewer than one third succeed in solving the problems in spite of the irregularities.

As is indicated in Table III, and by the typical answers which have been quoted, pupils who are unsuccessful manifest various reactions of confusion, perplexity, resentment, bluff, caution, humility, random guessing, and sincere attempts to solve problems in spite of insufficient data, or to use all of the data—some of which are irrelevant. Confusion and perplexity are shown by such expressions as "Too many words. I get all mixed up and can't make heads or tails of it," or by futile attempts to solve a problem leaving several uncompleted solutions. Humility is evinced in the confession, "I did not pay attention while the teacher was explaining this kind of a problem," or "I think I need to study more." Caution is shown by the pupil who seems to have had his suspicions aroused and is unwilling to attempt problems for which he is prepared. Resentment is expressed by, "Silly question!" and "How do you think I know? I've never seen the place or heard of it."

In many cases the pupil seems not to reflect. If the situation is unfamiliar he may guess at random, as in the typical answer 2 (a) page 24. Here the pupil juggles the results until he gets an answer which apparently satisfies him. If the situation is familiar, he may automatically identify and match up solutions after only a partial reading of the special problem, for example,

problem B, page 29, under both Superfluous and Insufficient Data, typical answers 1 (a) and (b), also typical answer 1 (c) to Problem A, page 28, under the same head. Considerable ingenuity is exercised by the pupil in the last mentioned attempt at a solution. The pupil selects data and formulates two equations which seem to represent the conditions of the problem, and which resemble second degree equations which he has solved by factoring and equating each factor to zero. He shows insight into the conditions of the problem and formulates equations with skill, but seems wholly oblivious to the fact quantities representing the areas of the proposed foundations for a house have been equated to quantities representing sums of money.

One fourth of these pupils are so habituated to the use of all of the data in a problem that they attempt to use all data, however irrelevant. One teacher reported, "The directions to the pupils are not sufficient to get them to exercise judgment about the data in a problem. Our pupils' polite conservatism leads them to believe that any problem in an examination or a text book can be solved." Another teacher remarked, "These problems would upset one of my pet theories. I tell the pupils they must always use all statements in a problem."

Problems which contain insufficient data offer more difficulty for these pupils than those which contain superfluous data. In these preliminary tests 25 per cent of the pupils attempt to solve problems in spite of the fact that essential data are missing, while but 10 per cent attempt to use superfluous data which are irrelevant, judging from items 3 (a) and (b) Table III. That superfluous information in a problem is more readily disregarded than are superfluous numbers is indicated in later parts of the investigation.² Superfluous information is regarded as more or less interesting or encumbering details in the reading of the problem. Numbers are considered quantities which must be used; "if otherwise, why should they be in the problem?"

The character of the pupils' responses is further analyzed in the study, which immediately follows, of the comparative success of pupils on special problems and on corresponding regular problems.

² See Section D, pages 34 and 36, problems under I and II; also Table XI in Section E, page 51.

It is plausible that pupils are confused or baffled, or that they resent problems of the special type, in many cases, because the problems are sprung on them unawares, and because they have not been taught to expect problems of their nature. Possibly if pupils were instructed to exercise discrimination in the choice of data, and if they occasionally met problems which require such discrimination, their confusion and exasperation would be reduced or even changed to enjoyment. The teaching experiments reported later in this study represent attempts to ascertain the effects of instruction along these lines.

D. COMPARISON OF SUCCESS OF PUPILS ON SPECIAL PROBLEMS WITH SUCCESS ON CORRESPONDING REGULAR PROBLEMS

In order to discover the effect of superfluous or insufficient data in a problem, it is necessary to know whether the pupils are familiar with the same type of problem when it contains only the necessary data. Accordingly, both regular and special problems are included in the tests. When a pupil solves a regular problem, it is assumed that he is familiar with the type of which both problems are examples. If he fails to solve the corresponding special problem, the failure may be due to (1) differences in the verbal content of the two problems, (2) differences in the numerical elements, (3) the presence in the special problem of superfluous data or the absence of essential data. No attempt has been made to separate the effects of these differences, but they are accounted for as follows:

The effects of differences in verbal content vary with individual pupils according to their individual experiences. Content which may be favorable to the success of one pupil may be unfavorable to the success of another. The chances are that, among a large number of pupils, the favorable and unfavorable influences of differences in content are equally balanced between corresponding problems. On the other hand, the effects of the presence of superfluous data or insufficient data in a problem remain constant for all pupils who are familiar with the type of problem.

It is desirable that corresponding regular and special problems differ in content in order that the similarity between them may not be so obvious that pupils may solve the special problem by analogy without having considered its special features. Such

instances were not uncommon in preliminary tests. Accordingly, corresponding problems were chosen which, while they differ in verbal content, both are phrased in content which is common to text books and courses of study. Attempts were made to keep the differences in content at a minimum, and to make the presence of superfluous data or insufficient data the primary differences.

In the same way attention may be directed to differences in the numerical content of the problems, for example, in the problems which follow:

(a) A stenographer spends $\frac{1}{3}$ of her monthly income for room and board, $\frac{1}{6}$ for clothes, and $\frac{1}{4}$ for other expenses. She saves \$30. What is her income?"

(b) "A clerk spends $\frac{1}{4}$ of his yearly salary for room and board, $\frac{1}{8}$ for clothing, and $\frac{1}{6}$ for other expenses. He saves \$880 which he invests in bonds bearing $4\frac{1}{2}$ per cent interest. What is his yearly salary?"

The two problems are similar in verbal content but are not identical. The second problem contains a superfluous phrase. The fractions and the whole numbers used in the problems vary, and the mixed number $4\frac{1}{3}$ occurs in the superfluous element of the second problem.

The differences in difficulty between using the fraction $\frac{1}{3}$ in the first problem and the fraction $\frac{1}{8}$ in the second, and the whole numbers \$30 and \$880 in the two problems, respectively, may constitute real difficulties for children in the elementary grades, but they offer no perceptible difference in difficulty for pupils who have advanced to the first year of high school mathematics. Such difficulties would occur in the solution of the equations rather than in the interpretation of the problem and the formulation of an equation which represents its conditions. The results of the tests show that the pupils who took them are more proficient in ability to solve equations than in ability to derive equations.

The diagrams which follow show that small percentages of the pupils fail to solve a problem after an equation has been derived. The final summary, in Figure 15, page 42, is representative of what happens in the separate tests. Of 678 pupils who were fully successful with the regular problems, 62 per cent completely failed on the corresponding special problem, 33 per cent were fully successful, and 5 per cent were partially

successful. Of 254 pupils who interpreted the problems correctly, only 13 per cent made some error in the mathematical computations.

In order to reduce the possible effects of differences in difficulty of different numerical elements, only numbers were used which are common to elementary arithmetic. The chances for difficulty are distributed indifferently among the regular and special problems.

A special problem, therefore, tests a pupil's ability to get a problem clearly in mind, and to select from a mass of data those which are necessary for its solution. A regular problem tests a pupil's familiarity with the type of problem, of which both the regular and special problem are examples, in so far as the problems are of equal difficulty. Otherwise, the regular problem predicts his ability to solve that type of problem after he has selected the essential data.

On the following pages are offered illustrations of corresponding special regular problems, together with the scores of the pupils who attempted to solve them.

The following figures show the comparative results for two or three of each of the special types of problems and corresponding regular problems of essentially the same types.

I. PROBLEMS WHICH CONTAIN SUPERFLUOUS NUMBERS VS. CORRESPONDING REGULAR PROBLEMS OF THE SAME TYPES

1. (a) A man can do a piece of work in 5 hours. His helper can do the same piece of work in 8 hours. How long will it take them to do the work if they work together?

(b) The capacity of a certain swimming pool is 133,000 gallons. It can be filled in 40 hours by one pipe which has 5 outlets, and in 60 hours by a second pipe which has only 3 outlets. It is desired to fill the pool as quickly as possible. How long should it take if the water runs through both pipes at the same time?

(b) Special Problem

(a) Regular Problem	Score	0	1	2	T
	0	122	0	1	123
	1	5	0	0	5
	2	42	7	17	66
	T	169	7	18	194

FIG. 1. Scattergram. Problems 1 (a) and (b), Superfluous Numbers.

The diagrams show the distribution of scores of two corresponding problems. A score of 2 indicates success, a score of 1 indicates partial success, a score of 0 indicates complete failure.

Horizontal rows show the distributions of the scores on the special problem of the pupils making each score on the regular problem. For example, the first row of Figure 1 may be read: Of 123 pupils who score 0 on the regular problem, 122 scored 0 on the special problem, 1 scored 2 on the special problem. The third row may be read: Of 66 who scored 2 on the regular problem, 42 scored 0, 7 scored 1, and 17 scored 2, on the special problem.

Similarly, the vertical columns show the distributions of the scores on the regular problem of the pupils making each score on the special problem. Column and row headed T give the totals.

2. (a) Mr. Moore decided to divide a sum of money among three investments. He invested one-third of it in oil stock, one-half of it in bank stock, and the remaining \$1000 in Liberty Bonds. How much did he invest?

(b) A man has invested a sum of money in three enterprises. One-half of it is invested in bonds paying 4 per cent interest, one-third in bonds paying 3 per cent interest, and the remaining \$2500 in bonds paying 5 per cent. How much has he invested?

(b) Special Problem

(a) Regular Problem	Score	0	1	2	T
	0	39	8	9	56
	1	15	1	1	17
	2	44	3	4	51
	T	98	12	14	124

FIG. 2. Scattergram. Problems 2 (a) and (b), Superfluous Numbers.

3. (a) When a six-foot pole casts a shadow $4\frac{1}{2}$ feet long, how long is the shadow of an eight-foot pole?

(b) A flag pole which is 12 inches in diameter at the base tapers to a diameter of 6 inches at the top. The shadow cast by the pole is 80 feet long. At the same time the shadow of a man who is 6 feet tall and is standing 30 feet from the pole is 8 feet long. How high is the flag pole?

(b) Special Problem

(a) Regular Problem	Score	0	1	2	T
	0	54	1	7	62
	1	2	0	2	4
	2	22	0	30	52
	T	78	1	39	118

FIG. 3. Scattergram.³ Problems 3 (a) and (b). Superfluous Numbers.

4. Summary for the Three Pairs of Problems.

(b) Special Problems

(a) Regular Problems	Score	0	1	2	T
	0	215	9	17	241
	1	22	1	3	26
	2	108	10	51	169
	T	345	20	71	436

$$r = -.123 \pm .04$$

FIG. 4. Scattergram. Summary for problems with Superfluous Numbers.

From Figure 4 it is seen that of 169 pupils who were fully successful on the regular problems, 108 failed to score on the special problems. Of 195 ($169 + 26$) who were partially or wholly successful on the regular problems, 120 ($108 + 22$) or 61.5 per cent failed on the special problems. Out of 436 pupils, 215 or 49 per cent failed to solve either the regular or the special problems, and are therefore judged to be unfamiliar with the types. Of 241 who failed to solve the regular problems, 17 or 7 per cent were fully successful, and 9 or 3.7 per cent were partially successful with the special problems.

II. PROBLEMS WHICH CONTAIN SUPERFLUOUS INFORMATION VS. CORRESPONDING REGULAR PROBLEMS OF THE SAME TYPES

1. (a) A piece of iron 52 inches long is to be cut into two parts so that one part is 4 inches longer than the other. What will be the length of each part?

³For interpretation of all diagrams see explanation page 35.

(b) The first appropriation for a library for Congress was made in 1800. The present Library for Congress was completed in 1897 at a total cost of six million dollars, the gold leaf on the dome alone costing \$3800. If the first appropriation had been \$1200 less than it was, it would have paid only for the gold leaf on the dome of the present building. What must have been the amount of the first appropriation?

(b) Special Problem

(a) Regular Problem	Score	0	1	2	T
	0	9		2	11
	1	1		1	2
	2	58	4	35	97
	T	68	4	38	110

FIG. 5. Scattergram. Problems 1 (a) and (b), Superfluous Information.

2. (a) A furniture dealer wishes to make a profit of 40 per cent on the selling price of a shipment of dining-room sets for which he paid \$156 each. At what price should he sell them in order to realize this profit?

(b) The pupils in a history class of fifty members have each agreed to buy a supplementary book. A book company is advertising a new book which has been adopted by many schools and is well liked. The selling price is \$2. It is printed on good paper and contains three hundred pages of reading matter. The principal can secure the books for $\frac{4}{5}$ of the selling price if he buys a large number of them. How much will each pupil have to pay for his book?

(b) Special Problem

(a) Regular Problem	Score	0	1	2	T
	0	8	2	1	11
	1	6	2	3	11
	2	54	3	45	102
	T	68	7	49	124

FIG. 6. Scattergram. Problems 2 (a) and (b), Superfluous Information.

3 (a) The area of the Caspian Sea is 180,000 square miles. If the area of Lake Superior were decreased by 12,000, the result would be equal to the area of the Caspian Sea. What is the area of Lake Superior?

(b) The elevation of Mt. Whitney in California, the highest recorded point in the United States, is 14,501 feet measured from sea level. The lowest point of dry land in the United States is Death Valley which also is in California. If 52 times the elevation of Death Valley be decreased by the elevation of Mt. Whitney the sum is zero. What is the elevation of Death Valley.

(b) Special Problem

(a) Regular Problem	Score	0	1	2	T
	0	18		5	23
	1	3	1	8	12
	2	20	14	30	64
	T	41	15	43	99

FIG. 7. Scattergram. Problems 3 (a) and (b), Superfluous Information.

4 Summary for the Three Pairs of Problems.

(b) Special Problems

(a) Regular Problems	Score	0	1	2	T
	0	35	1	8	44
	1	10	3	12	25
	2	132	21	110	263
	T	177	25	130	332

$$r = -.417 \pm .06$$

FIG. 8. Scattergram. Summary for Problems with Superfluous Information.

From Figure 8 it is seen that of 263 pupils who were fully successful on the regular problems, 132 failed to score on the special problems. Of 288 (263 + 25) who were partially or wholly successful on the regular problems, 142 or 49 per cent failed on the special problems. But 35 or 10.6 per cent failed on both and are therefore judged unfamiliar with the types. Of 44 who failed to solve the regular problems, 8 were fully successful and 1 was partially successful on the special problems.

III. PROBLEMS WHICH CONTAIN INSUFFICIENT DATA VS. REGULAR PROBLEMS OF THE SAME TYPE

1. (a) A father earns twice as much in a day as his son. The father worked 6 days and the son worked 5 days for which they received \$34. How much should each receive.

(b) A bus can carry 10 more passengers than an auto. To accommodate all the passengers arriving on a certain train, 7 autos and 2 buses will be needed. How many passengers are arriving?

(b) Special Problem

(a) Regular Problem	Score	0	1	2	T
	0	83	2	13	98
	1	10		1	11
	2	62	1	22	85
	T	155	3	36	194

FIG. 9. Scattergram. Problems 1 (a) and (b), Insufficient Data.

2. (a) A lawyer can afford to pay only \$40 a week for office help. He must pay the office boy \$5 a week, and he estimates the stenographer's work is worth two-thirds as much as the bookkeeper's. How much can he pay each?

(b) In 100 pounds of good corn fertilizer the amount of phosphoric acid exceeds the amount of nitrogen by 10 pounds, and the amount of potash should equal in weight the sum of the other two. How many pounds of each of these three plant foods would a fair crop of corn remove from an acre of ground?

(b) Special Problem

(a) Regular Problem	Score	0	1	2	T
	0	47		1	48
	1	15		3	18
	2	46		12	58
	T	108		16	124

FIG. 10. Scattergram. Problems 2 (a) and (b), Insufficient Data.

3. (a) A school room which seats 35 pupils has three times as many single desks as double desks. How many are there of each kind?

(b) A recipe for fruit punch calls for twice as many oranges as lemons, and proportional amounts of sugar, grape juice and water. If oranges are 35 cents a dozen and lemons are 25 cents a dozen, how many dozen of each must be used in making a punch at a total cost of \$4.75?

(b) Special Problem

(a) Regular Problem	Score	0	1	2	T
	0	63		2	65
	1	20		1	21
	2	30		2	32
	T	113		5	118

FIG. 11. Scattergram. Problems 3 (a) and (b), Insufficient Data.

4. Summary for the Three Pairs of Problems.

(b) Special Problems

(a) Regular Problems	Score	0	1	2	T
	0	193	2	16	211
	1	45		5	50
	2	138	1	36	175
	T	376	3	57	436

$$r = -.227 \pm .04$$

FIG. 12. Scattergram. Summary problems with Insufficient Data.

From Figure 12, it is seen that of 175 pupils who were fully successful on the regular problems, 138 failed on the specials. Of 225 (175 plus 50) who were partially or wholly successful on the regular problems, 183 (138 plus 45) or 83.3 per cent failed to score on the special problems. 193 or 44.3 per cent failed to score on either and are therefore judged unfamiliar with the types. Of 211 who failed to score on the regular problems, 16 were fully successful and 2 were partially successful on the special problems.

IV. PROBLEMS WHICH CONTAIN SUPERFLUOUS DATA BUT WHICH ALSO LACK ESSENTIAL DATA VS. REGULAR PROBLEMS

1. (a) The length of a tennis court is 42 feet more than the width and the perimeter is 228. What are the dimensions of the tennis court?

(b) A man had plans for a house of which the foundation was to be 12 feet longer than it was wide. Finding that it would cost \$18,000, which was more than he could afford, he decided to cut off 3 feet from the length and 2 feet from the width. In this way he lowered the cost to \$15,000. What were the dimensions of the foundation before and after making the changes?

(b) Special Problem

(a) Regular Problem	Score	0	1	2	T
	0	57	3	7	67
	1			1	1
	2	33	2	7	42
	T	90	5	15	110

$$r = -.502 \pm .07$$

FIG. 13. Scattergram. Problems 1 (a) and (b), Superfluous Data and Insufficient Data.

2. (a) The length of a rectangle is three times its width. The area is 48. What are the dimensions?

(b) The main waiting room in the Union Railway Station at Washington, D. C., has an area of 28,600 square feet. The length exceeds the width by 90 feet. How many people can be accommodated in this waiting room?

(b) Special Problem

(a) Regular Problem	Score	0	1	2	T
	0	48	1	18	67
	1			4	4
	2	10		18	28
	T	58	1	40	99

$$r = -.270 \pm .09$$

FIG. 14. Scattergram. Problems 2 (a) and (b), Superfluous Data and Insufficient Data.

V. SUMMARY OF SCORES ON ALL THE SPECIAL PROBLEMS
AND CORRESPONDING REGULAR PROBLEMS, IN SECTION D

(b) Special Problems

(a) Regular Problems	Score	0	1	2	T
	0	548	16	66	640
	1	77	4	25	106
	2	421	34	222	678
	T	1046	54	313	1434

$$r = -.231 \pm .01.$$

FIG. 15. Scattergram. Summary of scores on corresponding regular and special problems.

Of 784 pupils who scored 1 or 2 on the regular problems, 498 or 63.5 per cent failed to score on the special problems; 247 or 31.5 per cent were wholly successful. It is evident from these figures and per cents that pupils may be familiar with a type of problem, and may be able to solve problems of the type when supplied with only the necessary data, and yet be wholly unable to select from a mass of data in a problem of the same type those data which are necessary for its solution; when essential data are missing they fail to detect their absence.

A general inability of pupils to solve verbal problems is indicated by the large per cents who fail to solve either the special or the regular problems. Of the total number of 1424 responses, 548 or 38.5 per cent failed on both. As the test problems were chosen from the texts which are used in the classes tested, it appears that verbal problems commonly are omitted or neglected in teaching. Statements by the teachers further support this conclusion.

As has been indicated previously, a comparatively small per cent of the pupils who successfully interpret a problem and select the essential data are unsuccessful in completing the solution correctly. In Figures 10 and 11 no pupils scored partial success on the special problem. In the other figures the per cents of pupils who scored partial success on the special problem range from 0.84 per cent to 15 per cent. The summary of all of the

figures shows that but 5 per cent scored partial success. This may be due to the emphasis which teachers have placed upon the mathematical computations rather than upon the interpretation of verbal problems by the pupils.

The comparative success of pupils with regular and corresponding special problems is further shown in the data of the following section, in connection with a study of the extent to which the results just demonstrated are common among high school pupils.

E. EXTENT TO WHICH THE RESULTS ARE GENERAL AMONG
HIGH SCHOOL PUPILS

The extent to which the results previously reported are common among first-year high school pupils is indicated by the random sampling of high school classes from several states which was used in these studies. The following section shows:

1. The schools participating.
2. The per cent of all classes who solved each problem of each set.
3. The per cent of all classes who solved the special problems and the per cent who solved the corresponding regular problems.
4. The total per cent who solved the special problems, in those classes only where at least fifty per cent of the pupils solved the corresponding regular problems.
5. The distributions of the scores attained by six classes at Wadleigh High School of New York City, on the problems of Set VIII.
 - a. Total scores on 12 problems, 6 regular and 6 special.
 - b. Comparison of scores on 6 regular and 6 special problems.
 - c. Comparison of scores on 2 superfluous number problems *vs.* 2 corresponding regular problems.
 - d. Comparison of scores on 2 superfluous information problems *vs.* 2 corresponding regular problems.
 - e. Comparison of scores on 2 insufficient data problems *vs.* 2 corresponding regular problems.
 - f. The per cent of those who solve the regular problems who make scores of 1, 2, and 0, on the corresponding special problems.

TABLE IV

SCHOOLS WHICH PARTICIPATED IN THE PRELIMINARY TESTS

Number of Schools	16	Number of Classes	36
Number of States	9	Number of Pupils	1036

Name of High School	Location	Classes	Pupils
Mobile	Mobile, Ala.	2	49
Selma	Selma, Ala.	2	56
Ottawa Township	Ottawa, Ill.	1	27
Kokomo	Kokomo, Ind.	2	40
Atchison	Atchison, Kan.	2	55
Winfield	Winfield, Kan.	2	73
Luther W. Wright	Ironwood, Mich.	1	26
High Point	High Point, N. C.	1	30
Hackensack	Hackensack, N. J.	1	22
Bronxville	Bronxville, N. Y.	1	27
Erasmus Hall	Brooklyn, N. Y.	4	124
Yonkers	Yonkers, N. Y.	2	53
Titusville	Titusville, Pa.	2	45
Morris	New York City	3	91
Washington Irving	New York City	4	132
Wadleigh	New York City	6	206
Totals		36	1056

Per Cent of Pupils Who Solved Each Problem.—Problems 1 and 6 of each set, I to VI, are identical. They furnish a crude basis for comparing all classes on knowledge of simple verbal problems. Pupils generally were successful with problem 1, “A brick house is worth three times as much as a frame house. If both houses are worth \$12,240 how much is each worth?” Only 6.2 per cent of 726 pupils failed on this problem. Problem 6 proved to be more difficult: “Mr. Anderson wishes to enclose a rectangular field that is 20 rods wide. He wishes to make it as long as he can, using 214 rods of fencing. How long can he make it?” Of the same 726 pupils, 47.9 per cent failed to solve problem 6.

Problem 3 of each set contains superfluous data, and corresponds to regular problem 4. Problem 5 of each set contains insufficient data, and corresponds to regular problem 2.

Table V gives the per cent of pupils in each class who solved each problem correctly. The diagrams in Figure 16 represent the same results for the special test problems. In Figure 16 the total per cent who solved the problems of each set are represented, with corresponding problems arranged in pairs. The figures show that pupils are more successful with regular prob-

lems than with corresponding regular problems. The exceptions are problems 2 and 5 of Sets V and VI. In the former 60 per cent did not solve the regular problem; 52 per cent solved neither the regular nor the special problem. In the latter case the regular and special problems were equally difficult, 19 per cent solving each.

TABLE V
PER CENT WHO SOLVED EACH PROBLEM IN SETS I TO VII

Set	Class	No. of Pupils	Problems*					
			1	2	3+	4	5-	6
I	W	31	85%	24%	0%	0%	3%	24%
	Q	27	96	48	0	4	33	52
	T	33	100	61	39	88	50	56
	B	20	80	43	0	0	5	18
	I	26	85	73	8	44	13	50
	F	55	89	48	12	49	15	68
	Total	192	90	49	11	36	20	49
II	R	26	94	23	85	98	35	29
	X	35	91	27	51	89	3	34
	A	49	87	64	89	100	29	67
	Total	110	90	43	76	95	22	48
III	M	28	100	42	21	32	0	63
	Y	31	85	16	16	63	0	27
	G	32	73	20	25	16	3	51
	D	27	98	72	78	91	15	98
	Total	118	88	36	34	49	4	59
IV	N	34	100	66	40	96	22	49
	Z	35	96	20	24	97	14	20
	H	41	78	30	44	77	12	45
	Total	110	90	38	36	89	16	38
V	O	31	94	53	73	90	55	76
	U	32	91	47	38	66	41	25
	C	36	90	36	4	58	29	6
	Total	99	91	42	39	70	40	46
VI	P	31	97	26	50	90	19	66
	V	26	94	29	46	96	37	44
	E	40	94	9	30	83	8	63
	Total	97	94	19	41	88	19	58

*(+) Indicates superfluous data; (-), insufficient data. Corresponding problems; Sets I to VI, 3 vs. 4, 5 vs. 2; Set VII, 3 vs. 7, 5 vs. 9, 8 vs. 2, 10 vs. 4.

Set	Class	No. of Pupils	Problems*									
			1	2	3+	4	5-	6	7	8-	9	10+
VII	L	27	93%	56%	20%	93%	11%	89%	43%	17%	50%	59%
	K	22	75	25	18	73	5	77	25	0	11	11
	S	45	89	64	18	89	11	83	62	56	68	39
	J	30	83	70	8	87	23	93	63	15	70	63
	Total	124	86	57	16	86	13	86	52	27	54	44

* (+) Indicates superfluous data; (-), insufficient data. * Corresponding problems; Sets I to VI, 3 vs. 4, 5 vs. 2; Set VII, 3 vs. 7, 5 vs. 9, 8 vs. 2, 10 vs. 4.

Comparisons cannot be made of successes of different classes who solved different problems of which the relative difficulty has not been determined. Differences may be due to differences in ability of the classes or to differences in difficulty of the problems. It is evident, however, that failure to succeed with the special problems to the same degree that they succeed with regular problems of the same kinds, is common among all the classes tested. The per cents who are successful vary among classes. On problems containing superfluous data the per cents who are successful range from 0 in classes G and K to 89 in class A. In 25 out of 26 classes the per cents who are successful on regular problems exceed the per cents who are successful on corresponding special problems by differences ranging from 5 to 72. On problems containing insufficient data the per cents of successful solutions range from 0 in classes M to Y to 63 in class J. In 23 of 26 cases the per cents who are successful in regular problems exceeds the per cents who are successful on corresponding special problems by differences ranging from 6 to 60. Classes R and U were more successful with one of the special problems, and class E was equally successful with special and regular problems.

The relative successes of classes on different problems are indicated by the graphical representations in Figure 16 of the per cents of the total number of pupils solving each pair of corresponding problems who were successful on each problem.

Set VIII Given at Wadleigh High School. The whole story of a test is not told by percentages, or numerical figures representing central tendencies and variabilities of groups. Distribution tables represent to the eye what is less readily conveyed

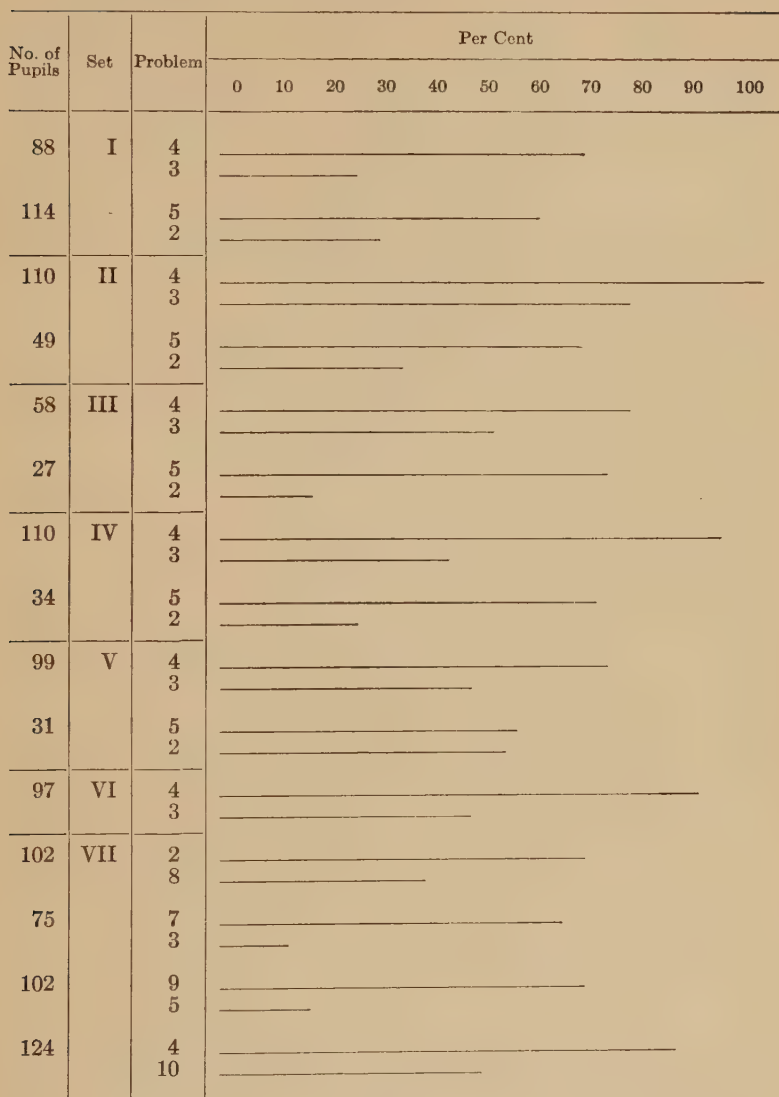


FIG. 16

PER CENT SOLVING REGULAR PROBLEMS *VERSUS*
CORRESPONDING SPECIAL

TOTAL PER CENTS FOR THOSE CLASSES IN WHICH A MINIMUM OF FIFTY
PER CENT OF THE PUPILS SOLVED THE REGULAR PROBLEMS

by these measures. The following tables give the distributions of the scores of 206 pupils in six classes at Wadleigh High School in New York City, on the test problems of Set VIII. Table VI shows the distributions of the scores of each class on the total of 12 problems, with a possible score of 24. In Tables VII to X the distributions of the scores on the different types of special problems are shown parallel to the distributions of the scores on the corresponding regular problems.

TABLE VI

DISTRIBUTIONS OF SCORES ATTAINED BY SIX CLASSES AT WADLEIGH HIGH SCHOOL ON PROBLEMS OF SET VIII

Scores	Class 1	Class 2	Class 3	Class 4	Class 5	Class 6	Total
24	2	--	--	--	--	--	2
23	--	--	--	--	--	--	--
22	1	--	--	--	--	--	1
21	--	--	--	--	--	1	1
20	2	2	--	1	--	1	6
19	--	--	--	--	3	2	5
18	6	2	--	4	1	--	13
17	1	5	2	2	1	--	11
16	3	3	1	4	2	3	16
15	6	3	3	2	3	2	19
14	1	3	1	3	5	3	16
13	3	1	4	2	8	1	19
12	3	3	5	4	3	5	23
11	1	1	7	4	4	3	20
10	1	1	3	3	1	7	16
9	1	2	1	--	1	2	7
8	5	--	3	1	2	3	14
7	1	--	1	--	2	3	7
6	1	2	2	--	--	3	8
5	--	--	1	--	--	1	2
4	--	--	--	--	--	--	--
3	--	--	--	--	--	--	--
2	--	--	--	--	--	--	--
1	--	--	--	--	--	--	--
0	--	--	--	--	--	--	--

<i>N</i>	206
<i>M</i>	12.9
<i>SD</i>	3.9
<i>SD_M</i>	.27

The final table of the group (Table XI) gives the per cent of individual pupils who were fully successful on a type of regular problem, who scored 2, 1, or 0 on the corresponding special problem, where 2 equals complete success, 1 equals partial success, and 0 equals complete failure.

TABLE VII

COMPARISONS OF THE DISTRIBUTIONS OF SCORES ON SIX REGULAR PROBLEMS
VERSUS SIX SPECIAL PROBLEMS

Scores	Group 1		Group 2		Group 3		Totals	
	Reg.	Sp.	Reg.	Sp.	Reg.	Sp.	Reg.	Sp.
12	17	2	10	--	10	--	37	2
11	12	--	11	--	13	--	36	--
10	15	1	13	--	15	--	43	1
9	7	1	14	--	11	1	32	2
8	6	5	7	5	9	2	22	12
7	3	3	3	1	8	2	14	6
6	2	13	8	9	5	5	15	27
5	3	5	--	3	--	4	3	12
4	1	8	--	17	2	8	3	33
3	--	7	1	4	--	11	1	22
2	--	12	--	16	--	17	--	45
1	--	3	--	7	--	8	--	18
0	--	6	--	5	--	15	--	26

<i>N</i>	206	206
<i>M</i>	9.84	3.49
<i>SD</i>	2.07	2.51
<i>SD_M</i>	.14	.18
<i>*SD_D</i>	3.25	

In Tables VII to XI the scores of the six classes have been combined into three groups of two classes each, which later composed the experimental groups.

* That the differences obtained in Tables VII to IX are real differences is demonstrated by finding the standard deviation of the difference of the mean scores between regular and special problems, according to the formula: $\sigma_x = \sqrt{\sigma_{reg}^2 + \sigma_{sp}^2}$.

TABLE VIII

DISTRIBUTIONS OF SCORES ON SUPERFLUOUS NUMBER PROBLEMS VERSUS
CORRESPONDING REGULAR PROBLEMS

Scores	Group 1		Group 2		Group 3		Totals	
	Reg.	Sp.	Reg.	Sp.	Reg.	Sp.	Reg.	Sp.
4	27	9	18	2	17	--	62	11
3	13	1	9	1	23	4	45	6
2	18	17	20	16	17	14	55	47
1	4	7	10	10	7	16	21	33
0	4	32	10	38	9	39	23	109
<i>N</i>							206	206
<i>M</i>							2.50	.91
<i>SD</i>							1.31	1.14
<i>SD_M</i>							.09	.08
<i>*SD_D</i>							1.73	

TABLE IX

DISTRIBUTIONS OF SCORES ON SUPERFLUOUS INFORMATION PROBLEMS VERSUS
CORRESPONDING REGULAR PROBLEMS

Scores	Group 1		Group 2		Group 3		Totals	
	Reg.	Sp.	Reg.	Sp.	Reg.	Sp.	Reg.	Sp.
4	44	19	49	22	68	14	161	55
3	2	2	14	3	3	4	19	9
2	16	22	4	27	2	23	22	72
1	2	3	--	4	--	5	2	12
0	2	20	--	11	--	27	2	58
<i>N</i>							206	206
<i>M</i>							3.63	1.96
<i>SD</i>							.79	1.57
<i>SD_M</i>							.06	.11
<i>*SD_D</i>							1.76	

* See note under Table VII.

TABLE X

DISTRIBUTIONS OF SCORES ON PROBLEMS WITH INSUFFICIENT DATA VERSUS CORRESPONDING REGULAR PROBLEMS

Scores	Group 1		Group 2		Group 3		Totals	
	Reg.	Sp.	Reg.	Sp.	Reg.	Sp.	Reg.	Sp.
4	41	--	41	2	34	--	116	2
3	8	--	15	--	14	--	37	--
2	7	11	8	10	19	10	34	31
1	7	7	3	--	3	2	13	9
0	3	48	--	55	3	61	6	164
<i>N</i>							206	206
<i>M</i>							3.18	.38
<i>SD</i>							1.10	.98
<i>SD_M</i>							.08	.07
* <i>SD_D</i>							.08	1.47

* See note under Table VII.

TABLE XI

PER CENT OF THOSE SOLVING A REGULAR PROBLEM WHO MAKE EACH SCORE ON THE CORRESPONDING SPECIAL PROBLEM

Scores on Sp.	Group 1			Group 2			Group 3			Totals		
	A*	B	C	A	B	C	A	B	C	A	B	C
4	20%	38%	4%	7%	35%	4%	--	19%	--	9%	30%	3%
3	--	3	2	4	5	--	8%	6	--	4	5	1
2	20	41	20	41	38	16	30	31	19%	29	36	18
1	15	5	10	7	6	--	20	7	--	15	6	3
0	45	13	64	41	16	80	42	37	81	43	23	75

A = Problems with superfluous numbers, B = problems with superfluous information, C = problems with insufficient data.

The totals of Table XI may be read thus: Reading from column A, of the pupils who were fully successful with the two regular problems 9 per cent scored 4 on the two corresponding problems containing superfluous numbers, 4 per cent scored 3, 29 per cent scored 2, 15 per cent scored 1, and 43 per cent scored 0. In this table a score of 4 is a perfect score on two problems.

F. SUMMARY

The effects of the introduction of problems which contain superfluous or insufficient data among types of problems with which pupils are familiar are shown by the data presented in Sections C, D, and E of this chapter. A brief resumé of the effects which have been stated at different points in the chapter follows.

In so far as the pupils who participated in the tests are representative of first-year high school pupils over the country, it may be concluded that pupils commonly are unaccustomed to problems in which selection must be made of the data to be used in solutions or in which pupils must find the necessary data. When such problems are introduced, the pupils being unaware of their special character, there is a variety of effects. As many as one third of the pupils are wholly unable to cope with the new situation; fewer than one third make satisfactory adjustments; the remaining pupils respond with varying degrees of success. The habit of attempting to solve a problem by using whatever data may be supplied without questioning their adequacy or relevance appears to be well established among pupils. Independent judgment and reflective thinking are evidenced by the smaller number who select the essential data or who demand additional data, even when they have not been accustomed to problems in which such selection must be made.

The scattergrams in Section D of this chapter represent objectively the relative success of pupils on regular and special problems of essentially the same types. Pupils who are familiar with methods of solving regular problems tend to be less successful with problems of the same types when they contain superfluous or insufficient data. This is true of special problems which contain superfluous numbers, superfluous information, insufficient data, or problems containing both superfluous and insufficient data.

Failure to succeed with the special problems to the same degree that they succeed with regular problems of the same types is common among the pupils in the classes tested. In twenty-three out of twenty-six cases the per cents who were successful on regular problems exceeded the per cents who were

successful on the special problems, by decisive majorities. In the foregoing tables thirty classes were reduced to twenty-six by combining small classes in the same schools. These twenty-six classes represent a random sampling of high school pupils from widely separated sections of the country, and from various types and sizes of schools.

The results of the tests in six classes at Wadleigh High School, reported in Tables VI to XI, indicate that the conditions found in the foregoing schools also prevail to a marked degree at Wadleigh High School. More precise measurement of the effects of the introduction of special problems are reported in subsequent sections of this study.

CHAPTER IV

DESCRIPTION OF EXPERIMENTS

THE EFFECTS OF APPROPRIATE INSTRUCTION ON THE ABILITIES TO SELECT ESSENTIAL DATA AND TO DETECT ABSENCE OF DATA. THE EXTENT TO WHICH SUCH ABILITIES ARE TRANSFERRED FROM PROBLEMS IN MATHEMATICS TO PROBLEMS IN OTHER SUBJECTS.

In the preceding chapter it was shown that pupils generally do not have the habits of selecting and demanding essential data. The next question is whether these abilities and habits can be developed by suitable instruction, and if so, how much instruction and drill are required to accomplish this end. This question was studied in a one-group experiment conducted by a teacher of first-year algebra in a middle-western high school, under the direction of the writer. With a limited amount of instruction, which is defined later, the pupils in two classes showed marked improvement, establishing the fact that a small amount of specific instruction will cause pupils to improve markedly in these functions.

The question of whether such habits when developed in algebra classes can be carried over to similar situations in other subjects, was then added, and the two questions were made the subject of experiments more highly controlled and precise. In all cases instruction and drill were followed by increases in ability to select essential data and to detect the absence of data when they are lacking from problems in algebra. The possibility of transfer of the abilities and habits to problems in other subjects of study, was established. The results of these studies are here reported in detail.

A. ONE-GROUP EXPERIMENT—ATCHISON, KANSAS

The pupils were forty-five boys and girls, members of two of four classes who were completing the first year of algebra in the

Atchison High School. The classes were taught by the same teacher. Ford and Ammerman's *Beginning Algebra* was the text.

The tests. The initial test was Set I, previously described, which contains six problems, two of which are special problems and two corresponding regular problems. The final test consisted of twelve problems, among which were six problems which were considered by five competent judges to be the equivalents of the six problems of the initial test. Comparisons were made between results on the six corresponding problems only.

Instruction and Testing. Late in the spring term the initial test was given without warning as to the character of the special problem. Following this test the pupils were given instruction concerning problems which contain superfluous and insufficient data. The instruction was limited to (1) explanation of the character of the special problems, with (2) five illustrations of each type, and (3) a miscellaneous set of special and corresponding regular problems. Care was exercised not to discuss the initial test. The special instruction was interspersed among the regular work of the class. After a period of two weeks the final test was given, again according to specific directions supplied by the writer. All papers were scored by the writer by the standards described on pages 23 and 24.

The Results. The mean score of the classes on the *initial* test was 5.82 points out of a possible 12. In the final test the mean was raised to 8.71. The distribution of the scores is shown below. Improvement is measured in terms of the mean of the changes of the scores of the pupils between the initial and final test. The results are given separately for scores on (1) the total problems of tests, (2) the special problems, and (3) the corresponding regular problems. There was a mean gain of 3.13 on the total scores. The greater portion of the gain, or 2.42, was on the special problems. The small improvement on the corresponding regular problems is explained by the highest scores made on these problems in the initial test, as are shown in the distribution in Table XII.

Improvement is the result of several factors; the practice effect of the initial test, explanation of the character of the special problems, and drill on the practice problems, all of which may be classified as instruction. The results indicate that only a

TABLE XII
IMPROVEMENT OF PUPILS IN ATCHISON HIGH SCHOOL
A. AVERAGE GAINS

	Total Scores	Scores on Special Problems	Scores on Corresponding Regular Problems
<i>M</i>	3.13	2.42	.49
<i>SD</i>	2.52	1.29	1.57
<i>SD_M</i>	.38	.19	.23

B. DISTRIBUTION OF SCORES

Scores	Total		2 Special Problems		Corresponding Regulars	
	Initial	Final	Initial	Final	Initial	Final
12	--	11	--	--	--	--
11	--	1	--	--	--	--
10	3	7	--	--	--	--
9	4	6	--	--	--	--
8	7	7	(4	a per-	fect score)	--
7	3	3	--	on two	problems	--
6	10	6	--	--	--	--
5	4	--	--	--	--	--
4	8	2	--	21	12	15
3	--	--	--	3	2	6
2	3	2	10	17	18	15
1	1	--	3	1	3	2
0	2	--	32	3	10	7
<i>M</i>	5.82	8.71	.51	2.84	2.07	2.44

C. DISTRIBUTION OF CHANGES IN INDIVIDUAL SCORES BETWEEN INITIAL AND FINAL TESTS. (MATHEMATICAL TESTS ONLY)

Change	Total Problems (12)*	Special Problems (2)*	Corresponding Regular Problems (2)*
10	1	--	--
9	--	--	--
8	--	--	--
7	1	--	--
6	8	--	--
5	2	--	--
4	7	14	2
3	6	4	2
2	13	19	10
1	2	3	3
0	1	5	11
- 1	2	--	5
- 2	1	--	4
- 3	--	--	2
- 4	1	--	--
<i>M</i>	3.13	2.42	.49

* Indicates the number of problems.

small amount of instruction is necessary to teach pupils to select data and to demand sufficient facts for the solution of algebra problems. It is not wholly that pupils do not have the ability to think in this field, but that they are not accustomed to problems in which they are expected to exercise judgment in choice of data, and probably they have been trained to use all data.

That this limited amount of instruction is not sufficient to insure correct thinking by all, is demonstrated by the fact that many pupils made erroneous judgments in the final test. Having been told that problems sometimes contain superfluous or insufficient data, they leaped to the conclusion that any unusual or difficult problem was one of the special type.

The results of this experiment were more fully tested in experiments now to be described.

B. EXPERIMENT IN WADLEIGH HIGH SCHOOL, NEW YORK CITY

The Plan. To test the results of previous studies, a series of controlled experiments was conducted in Wadleigh High School, with the coöperation of the head of the department and the teachers of first-year mathematics. The plan was to find three groups of pupils who were equivalent as groups, in general intelligence, amounts and conditions of training, mathematical standing, and ability to solve regular verbal problems. All groups were given initial tests (1) of their ability to solve algebra problems containing superfluous and insufficient data, and (2) of their ability to solve similar problems in other subjects which they were studying.

The three groups were then given different amounts and kinds of instruction on the special problems. After four weeks of instruction the pupils were tested again by algebra and non-mathematical tests, and comparisons were made of the improvement of the several groups in ability to solve special algebra problems, and of the indications of transfer of these abilities to the non-mathematical problems.

The Algebra Tests—Initial and Final. The tests used in these experiments were forms of Set VIII which was derived from the preliminary tests. The initial test, or form Alpha, consists of twelve problems, six of which are special and six are regular problems. Two of the special problems contain superfluous data

which are in the form primarily of verbal information; two contain superfluous data which are primarily numbers; two contain insufficient data. The test is divided into two parts, which were given on successive days.

The final test, or form Beta, was made as nearly as possible the equivalent of form Alpha in the number and types of problems. The problems of the two tests correspond in sentence structure, mathematical principles, and computations. They differ in the subjects of the problems. For example, see problem 6 and problem 3 of the two forms below. Complete forms of the tests are given in the Appendix.

The reliability of the tests is measured by the correlation of the scores of 62 pupils of the control classes on Alpha and Beta forms. A period of four weeks and three days had intervened between the tests, during which the pupils had no special instruction. The coefficient of correlation is $.84 \pm .047$.

PROBLEM 6, ALPHA

The members of a history class of fifty pupils have each agreed to buy a supplementary book. A book company is advertising a new book which has been adopted by many schools and is well liked. The selling price is \$2.00. It is printed on good paper, is well bound and contains three hundred pages of reading matter. The principal can obtain the books for the class for $\frac{4}{5}$ of the selling price, if he buys a large number of them. How much should each pupil pay for his book?

PROBLEM 6, BETA

The girls of the basket-ball squad consisting of ten members have decided to buy new uniforms for themselves. The Spalding Company advertises a uniform which the girls all like, and which sells for \$10.00. The material is of very good quality and the uniforms are well made. The colors are blue and gold. The athletic director can obtain the lot of ten uniforms for $\frac{4}{5}$ of the selling price, if cash is paid. How much will each girl have to pay for her uniform?

PROBLEM 3, ALPHA

A bus can carry 10 more passengers than an auto. To accommodate all the passengers arriving on a certain train 7 autos and 2 buses will be needed. How many passengers are arriving?

PROBLEM 3, BETA

In order to accommodate all the pupils who went on a sleighing party last winter, 5 large sleighs and 3 small sleighs were needed. Each large sleigh carried twice as many persons as a small sleigh. How many pupils were there in the party?

The Non-Mathematical Tests. The pupils were studying many parallel subjects, and not all were studying the same. Since it was impractical to give several different tests, it was decided to give a reading test,¹ the subject matter being selected from American history, science and general information. It was planned to make the test consist of simple questions based on paragraphs of reading matter. Three types of paragraphs were desired:

1. Paragraphs which contain all the essential data and little more. Pupils should be able to reach valid conclusions by considering and using all the data.
2. Paragraphs which contain more data than are needed. In order to reach valid conclusions the pupils must sift the data and make selections.
3. Paragraphs which contain insufficient data for answering the questions. Pupils may reach fallacious conclusions if they do not detect the omissions and do not demand all the necessary facts.

Paragraphs which seemed to meet these conditions were chosen from standardized tests,² the texts used by the pupils, and current periodicals. These paragraphs were submitted to a group of ten graduate students and professors of education with the request that they select and rate those which most nearly represented the desired types. Eighteen paragraphs, or six of each of the three types, were chosen. They were roughly standardized by use in three classes in White Plains, New York, and two classes in New York City, totaling two hundred pupils.

¹ The use of a reading test as a measure of ability to solve non-mathematical problems is supported by the following quotation from an article, "Reading as Reasoning" by E. L. Thorndike, *Jour. Ed. Psych.*, June, 1917:

"The act of answering simple questions about a simple paragraph includes all the features characteristic of typical reasoning. . . . In correct reading each word produces a correct meaning, each such element of meaning is given weight in comparison with others, and the resulting ideas are examined and validated to make sure that they satisfy the meaning set or adjustment or purpose for whose sake the reading is done. Reading may be wrong or inadequate because of wrong connections with wrong words singly, because of over potency or under potency of elements, or because of failure to test the ideas produced by the reading as provisional, and so inspect and welcome or reject them as they appear.

"Understanding a paragraph is like solving a problem in mathematics. It consists in selecting the right elements of the situation and putting them together in the right relations, and also with the right amount of weight or influence or force for each. The mind is assailed as it were by every word in the paragraph. It must select, repress, soften, emphasize, correlate, and organize, all under the influence of the right set or purpose or demand.

² Van Wagenen, American History Scales and Thorndike-McCall Reading Scale.

The final form of the test, with the standard answers used in scoring, is reproduced in the Appendix.

The same form was used for the initial and the final tests. The first was given two weeks in advance of the initial algebra test and the second was given after the final algebra test. Thus seven weeks had elapsed between the first and second uses of the test.

A measure of the reliability was obtained by correlating the scores on the two tests of 72 pupils of the control classes. The coefficient of correlation is $.73 \pm .037$.

The Pupils. The pupils who were the subjects of this experiment were the members of six beginning mathematic classes, originally 240 in number. After eliminating absentees, the number whose records were used was reduced to 195. The six classes were selected from among ten classes, by the head of the department, as those which were most nearly uniform in age, previous training, general ability, and mathematical standing.

Beginning mathematics in Wadleigh High School is mixed algebra and geometry. It is offered to pupils only after they have been in high school one half-year, and is an elective study. The pupils were therefore girls who had elected mathematics in their third half-year of high school and were completing their second half-year of mathematics.

Four experienced teachers were in charge of the six classes. Uniform teaching conditions were insured by the organization and supervision of the department. The head of the department, who is the author of the text book used, closely supervises the instruction and gives uniform examinations periodically. Teachers who enter the system are expected to familiarize themselves with the methods of the department by regularly observing other teachers in the system. During the course of the experiment the writer was in close touch with the teachers, meeting them in conference and visiting the classes.

Instruction and Drill. 1. Two of the classes under two teachers were given special instruction in algebra problems exclusively. These classes will be referred to as the *Math. Group*. Each week for four weeks the pupils were given a set of special problems along with the regular work of the class. The first was a set of five problems in which data were missing. The second was a set of five problems which contained superfluous

data. The third was a set of five problems giving practice in analyzing problems for essential elements. The fourth was a set of sixteen miscellaneous problems made up of regular and special problems of different types.

Instruction on the special problem occupied less than fifty minutes a week. To insure uniformity of instruction, the lessons and method of presenting them were outlined for the teachers by the writer, and problems were issued to each pupil on mimeographed sheets. The solutions were collected and checked by the teachers, and a complete record of each pupil's participation was kept. The lessons used are given, in full, in the Appendix.

2. Two classes under one teacher were given instruction and drill identical with that of the Math. Group, with the addition that the general character of the special problems was emphasized. Pupils were told that the abilities to detect the absence of missing data and to distinguish and select essential data are of importance not only in mathematics but in other subjects and outside of school. When different types of special mathematical problems were presented, appropriate illustrations were given of similar problems in non-mathematical situations. Care was exercised not to refer to the reading test and not to use any of the problems of that test as illustrations. These classes will be referred to as the *Non-Math. Group*.

3. Two other classes under one teacher, served as the control classes. They took the initial and final tests, but received no instruction on special types of problems. They were given the same number of regular verbal problems as the Math. Group and the Non-Math. Group had of special and regular problems, during the period of the experiment. From these classes the *Control Group* were selected.

The Process of Determining Equivalence among Groups. Since pupils could not be shifted from one class to another, it was necessary first to choose classes which were approximately equivalent and later to select from them smaller groups which were equivalent. The first choice was made after the experimenter had become acquainted with conditions in the school on the basis of information available in the school records. Exact pairing or matching was postponed until the close of the experiment.

No one basis is entirely satisfactory for equating groups. It would seem desirable to equate them on many bases, as chronological age, mental age, year in school, home environment, race, mathematical standing, status in the experimental trait. Such equating is impractical because with every increase in the number of bases employed the number of pupils who can be matched satisfactorily is reduced. Therefore (a) knowledge of subject matter and (b) general intelligence were selected as two significant bases for equating pupils on possibility for growth in the experimental traits. Measures of the former were the pupils' marks in (a) the uniform algebra examination and (b) the teachers' mid-term marks. Measures of intelligence were the scores on the Terman Mental Ability Group Test.

The Uniform Algebra Examination is a general examination over the work of the half-year. It was given as a part of the regular school routine, to all classes alike, immediately before the experiment was begun. The questions were prepared by the head of the department of mathematics, and all papers were marked by a central committee of teachers according to the same standard. These conditions make the examination a uniform basis for comparing pupils and classes in respect to their knowledge of mathematics at the beginning of the experiment.

The Teachers' Marks were the reports of the teachers given at the middle of the term during which the experiment was carried on. They were based upon the pupils' daily work, written tests, and teachers' estimates. That these marks are less subject to the eccentricities ordinarily found in individual standards and judgments, is indicated by their correlation of .53 with the uniform examination.

The Terman Mental Ability Group Tests were administered and scored by a trained psychologist and her assistant who are members of the high school staff. Standard procedure for giving and scoring the tests was followed.

A criterion with which to correlate the three bases of equivalence was needed. A measure of ability was desired to make progress under mathematical instruction. Two possible measures were available: (1) the progress made by the control group during the brief period of the experiment, and (2) the progress of the pupils in all the groups from the time they

began to study algebra, as measured by the initial test. The latter was chosen as the more reliable criterion, because of the larger numbers of pupils involved and the longer periods of growth.

The significance of the three bases was determined from their correlations with the initial algebra test. The scores of 195 pupils who participated in all of the tests were used. All show substantial positive correlations. The intercorrelations among the bases show high correlation between the Uniform Examination Marks and the Teachers' Marks, and relatively low correlation between each of them and the Terman Mental Ability Test scores.

	Initial Test	Uniform Examina- tion	Teachers' Marks	Terman Test
Initial Algebra Test	1.00	.50	.47	.44
Uniform Examination.....	.50	1.00	.53	.22
Teachers' Marks	.47	.53	1.00	.19
Terman Ability Test	.44	.22	.19	1.00

To avoid equating on three separate bases, a composite score was computed for each pupil, made up of the uniform examination marks, the teachers' mid-term marks, and the Terman Mental Ability Test scores. This gives approximate if not actual equivalence for each trait. The procedure for computing a composite score is described by McCall in *How to Experiment in Education*, pages 51 to 55. This first step was to compute the arithmetical mean and the standard deviation from this mean for each of the series of scores. The second step was to determine what weight should be given to each element of the composite score. From the correlations among them it was judged that the Uniform Examination and the Teachers' Marks should be given equal weight, and the Terman Test scores should be given weight equal to the two others combined. The third step was to choose a multiplier by which each of the scores of the respective series should be multiplied in order to make their respective variabilities equal, and at the same time give the desired weight to each element. The final step was to multiply

the respective series of scores by the number chosen, and to add the weighted scores for each pupil.

Multiplying (b) and (c) below by 1.62 and 2.42 respectively, makes their standard deviations equal to that of (a). Multiplying (a) by 2 gives the Terman Mental Ability Test a weight twice as great as that of (b) or (c), and equal to their sum.

	Standard Deviation	Multiplier	Derived Standard Deviation
a. Terman Test	30.73	2	61.46
b. Uniform Examinations	19.02	1.62	30.81
c. Teachers' Marks	12.68	2.42	30.69

DISTRIBUTION OF COMPOSITE SCORES OF 195 PUPILS

Scores	Frequency
720-739	2
700-719	--
680-699	1
660-679	4
640-659	6
620-639	7
600-619	10
580-599	6
560-579	10
540-559	16
520-539	18
500-519	15
480-499	15
460-479	17
440-459	14
420-439	15
400-419	11
380-399	9
360-379	7
340-359	5
320-339	5
300-319	2
<i>M</i>	598.9
<i>SD</i>	99.5
<i>SD_M</i>	7.1

From the larger groups which had served as experimental groups now were selected smaller groups of pupils whose composite scores were matched in such ways as to make three groups

alike in arithmetical means and standard deviations from the means. Equivalence means that groups shall have like central tendencies and variabilities. It does not require that individual scores shall be exactly matched, but exact matching is desirable when possible. The following groups were obtained: (1) Three groups of 45 pupils each in which the individual scores are approximately matched; (2) three equivalent groups of 25 pupils each in which the individual scores are closely matched.

A discrepancy in matching pupils from different groups is possibly due to the fact that overlapping among groups may be exaggerated through inadequacies and inaccuracies or unreliability of the measurements. The effect of more adequate and more accurate measurements would be to reduce the variability of

TABLE XIII

COMPOSITE SCORES OF THREE EQUIVALENT GROUPS VERY CLOSELY MATCHED

G* Math. Group	E Non-Math. Group	F Control Group
667	670	668
657	658	646
629	624	628
617	617	617
602	601	606
579	579	577
558	561	555
547	551	549
531	533	534
529	531	527
521	522	521
509	513	512
504	508	508
503	501	503
491	491	489
489	482	487
478	476	478
461	451	452
442	443	444
430	439	435
428	426	427
422	416	424
417	416	410
410	409	410
374	363	371
<i>N</i> 25	25	25
<i>M</i> 511.8	511.24	511.12
<i>SD</i> 79.1	80.6	79.0
<i>SD_M</i> ... 15.8	16.1	15.8

* The letters G, E, F will be used in this and later tables to designate the Math., Non-Math. and Control groups, respectively.

each group and therefore to reduce overlapping. Theoretically the scores in the overlapping areas which belong to the group having the higher mean are slightly better than they appear to be; scores which belong to the group having the lower means are slightly poorer than they appear. Unless correction of the scores for such error is made, a group with a higher mean may have an advantage over another group with which it is compared.

In this case the original Control Group has the highest mean of 534.2, and the original Non-Math. Group has the lowest mean of 492.9. The mean of the original Math. Group is 504.3. Thus the Control Group may have an advantage over the other two groups, and the Math. Group may have an advantage over the Non-Math. Group. If this advantage exists it strengthens the reliability of the gains made by the Math. and Non-Math. groups, over those of the Control Group, and the gain made by the Non-Math. Group over that of the Math. Group.

The results of the tests for the Math., Non-Math., and Control groups are summarized in the following tables. A record of the initial and final scores, and the gain or loss of each individual pupil has been retained and is open to inspection by interested persons. Table XV (A) gives the distribution of the changes in individual scores between the initial and final scores. Tables XV (B) and XVI give the mean gains of equivalent groups, and relative gains among comparable groups.

*Measures Used Throughout the Computations.*³—The change of each group between the initial test (*IT*) and the final test (*FT*) in all cases has been measured by the arithmetical mean (*M*) or average of the several changes of the individuals of the group. Variability of a group is indicated by the standard deviation (*SD*) from the mean of the individual changes. The reliability of the mean is indicated by standard deviation of the mean (*SD_M*), computed by the formula, $SD_M = \frac{SD}{\sqrt{N}}$, where *N* equals the number of individuals.

In comparisons of one group with another, the difference (*D*) between them is found by subtracting the mean change on one group (*M_x*) from the mean change of the other (*M_y*). The reliability of this measure of comparison, is indicated by the standard deviation of the difference (*SD_D*), computed by the formula:

³ McCall, W. A., *How to Experiment in Education*.

TABLE XIV

DISTRIBUTION OF CHANGES IN INDIVIDUAL SCORES BETWEEN INITIAL AND FINAL TESTS AT WADLEIGH HIGH SCHOOL FOR MATH., NON-MATH. AND CONTROL GROUPS

A. MATHEMATICAL TESTS

Change	Total Problems (12)			Regular Problems (6)			Special Problems (6)		
	Groups			Groups			Groups		
	G*	E	F	G	E	F	G	E	F
12	1	--	--	--	--	--	--	--	--
11	1	--	--	--	--	--	--	--	--
10	1	1	--	--	--	--	2	1	--
9	2	1	1	--	--	--	--	2	--
8	2	5	--	--	--	--	1	3	--
7	4	4	--	--	--	1	2	2	--
6	6	3	--	1	--	--	1	3	--
5	1	5	--	1	--	--	4	4	--
4	2	3	1	1	--	1	4	5	1
3	3	--	1	5	1	--	8	2	2
2	1	3	4	6	2	2	2	2	4
1	1	--	5	5	5	2	1	1	4
0	--	--	5	2	8	12	--	--	9
- 1	--	--	4	4	3	3	--	--	2
- 2	--	--	2	--	2	2	--	--	2
- 3	--	--	2	--	--	1	--	--	1
- 4	--	--	--	--	--	--	--	--	--
- 5	--	--	1	--	--	--	--	--	--
M	6.2	5.9	.6	1.7	.52	.08	4.5	5.4	.52

B. READING TESTS

Change	Total Problems (18)			Adequate Data (6)			Superfluous Data (6)			Insufficient Data (6)		
	Groups			Groups			Groups			Groups		
	G	E	F	G	E	F	G	E	F	G	E	F
9	--	1	--	--	--	--	--	--	--	--	--	--
8	1	1	--	--	--	--	--	--	--	--	--	--
7	--	2	--	--	--	--	--	--	--	--	--	--
6	1	--	--	--	--	--	--	--	--	--	--	--
5	--	4	--	--	--	--	--	--	--	1	--	--
4	4	3	1	--	1	--	--	1	--	1	1	--
3	4	6	3	2	1	1	--	--	1	3	3	1
2	7	2	1	3	7	--	2	4	--	5	11	1
1	6	2	9	9	4	8	10	3	8	6	7	7
0	1	3	5	7	9	10	11	4	8	7	3	13
- 1	1	1	2	4	3	4	2	9	8	2	--	2
- 2	--	--	3	--	--	1	--	4	--	--	--	1
- 3	--	--	1	--	--	1	--	--	--	--	--	--
M	2.4	3.5	.52	.68	.88	.08	.48	.92	.12	1.3	1.7	.32

* G, E, F will be used in this and subsequent totals to designate Math., Non-Math. and Control groups, respectively.

$SD_D = \sqrt{(SD_{M_x})^2 + (SD_{M_y})^2}$. The reliability is further indicated by the experimental coefficient (*EC*), which represents the degree of certainty that the true difference is above zero if the obtained difference is positive, or below zero if the obtained difference is

negative. The formula is $EC = \frac{D}{2.78 SD_D}$. An experimental coefficient of 1.0 means that we can be practically certain that the true difference is above zero, or that the approximate chances are 369 to 1. An *EC* of .5 means half certainty, while an *EC* of 2.0 means double certainty that the true difference is above zero.⁴

TABLE XV

MEAN GAINS OF EQUIVALENT MATH., NON-MATH. AND CONTROL GROUPS
ON ALGEBRA AND READING TESTS

A. ALGEBRA TESTS

Group	Measures	Total Scores	Regular Problems	Special Problems
Math. G	<i>M</i>	6.24	1.72	4.52
	<i>SD</i>	2.7	1.80	2.30
	<i>SD_M</i>	.54	.36	.46
Non-Math. E	<i>M</i>	5.92	.52	5.40
	<i>SD</i>	2.13	1.30	2.35
	<i>SD_M</i>	.43	.26	.47
Control F	<i>M</i>	.60	.08	.52
	<i>SD</i>	2.43	2.19	1.63
	<i>SD_M</i>	.49	.44	.33

B. READING TESTS

Group	Measures	Total Scores	Adequate Data	Superfluous Data	Missing Data
Math. G	<i>M</i>	2.44	.68	.48	1.28
	<i>SD</i>	1.86	1.12	.76	1.48
	<i>SD_M</i>	.37	.22	.15	.30
Non-Math. E	<i>M</i>	3.48	.88	.92	1.68
	<i>SD</i>	2.63	1.28	1.55	.98
	<i>SD_M</i>	.53	.26	.31	.20
Control F	<i>M</i>	.52	.08	.12	.32
	<i>SD</i>	1.70	1.16	1.01	.94
	<i>SD_M</i>	.34	.23	.20	.19

M, the arithmetical mean, shows the individual gain.

SD, the standard deviation from the mean, is used as a measure of variability of each group. *SD_M*, the standard deviation of the mean, is a measure of the reliability of the mean.

⁴By multiplying any *SD*, *SD_M*, or *SDD*, by .6745, it may be transmuted into *PE*, *PEM*, or *PED*, respectively.

TABLE XVI

COMPARISONS OF MEAN GAINS AMONG MATH., NON-MATH. AND CONTROL GROUPS OF ALGEBRA AND READING TESTS

A. ALGEBRA TESTS

Groups	Measures	Total Scores	Regular Problems	Special Problems
G vs. F Math. vs. Control	M (G)	6.24	1.72	4.52
	M (F)	.60	.08	.52
	D	5.64	1.64	4.00
	SD_D	.73	.57	.55
	EC	2.79	1.04	2.60
E vs. F Non-Math. vs. Control	M (E)	5.92	.52	5.40
	M (F)	.60	.08	.52
	D	5.32	.44	4.88
	SD_D	.65	.51	.57
	EC	2.96	.31	3.07
E vs. G Non-Math. vs. Math.	M (E)	5.92	.52	5.40
	M (G)	6.24	1.72	4.52
	D	-.32	-1.20	.88
	SD_D	.69	.44	.66
	EC	-.17	-.98	.48

B. READING TESTS

Group	Measures	Total Scores	Adequate Data	Superfluous Data	Missing Data
G vs. F Math. vs. Control	M (G)	2.44	.68	.48	1.28
	M (F)	.52	.08	.12	.32
	D	1.92	.60	.36	.96
	SD_D	.50	.32	.25	.35
	EC	-1.37	.67	.51	.98
E vs. F Non-Math. vs. Control	M (E)	3.48	.88	.92	1.68
	M (F)	.52	.08	.12	.32
	D	2.96	.80	.80	1.36
	SD_D	.63	.35	.37	.38
	EC	1.70	.83	.78	1.77
E vs. G Non-Math. vs. Math.	M (E)	3.48	.88	.92	1.68
	M (G)	2.44	.68	.48	1.28
	D	1.04	.20	.44	.40
	SD_D	.64	.34	.35	.36
	EC	.58	.21	.46	.40

M , the mean gain. D , difference in gains or changes; SD_1 , standard deviation of the difference; and EC , the experimental coefficient.

Results of the Algebra Tests.—Both the Math. and the Non-Math. groups showed positive gains in the total scores of 6.24 and 5.92, respectively. The Control Group showed a slight but

positive gain of .60. This last gain represents the change which should be attributed to normal growth and practice effect of the initial test for all the groups. When the gain of the Control Group is subtracted from the gains of the Math. and the Non-Math. groups the mean net gains for these groups become 5.64 and 5.32, respectively.

It should be recognized, of course, that this gain is due in part to attraction of attention to the special problems, and in part to practice in selecting data and detecting their absence. Pupils readily learned to expect problems of the novel types and became cautious about drawing conclusions without first having scrutinized the data. Having been made aware of the types of problems, pupils continued to show improvement in the special functions.

That these gains are due largely to changes in scores on the special problems rather than on the regular problems is indicated by the separate scores on these problems. The net mean gains of the Math. and Non-Math. groups on the six regular problems, respectively, are 1.64 and .44 as opposed to 2.60 and 4.88 on the six special problems.

The conclusions are that, for these groups, training produced improvement in the abilities and habits of choosing relevant data and detecting the absence of essential data in algebra problems. The time devoted to special training was limited to less than 50 minutes a week for four weeks. It is reasonable to believe that more training would produce increased improvement in skill in selecting the pertinent data and in detecting the absence of missing data.

Results of the Reading Tests.—All groups showed positive gains on the reading tests. The Control Group shows a slight but positive gain of .52. The Math. and Non-Math. groups gained 2.44 and 3.48, respectively. After subtracting the gains of the Control Group the net gains are 1.92 and 2.96, which may be attributed to the special training of these groups. In all cases greater gains were made in ability to answer questions on paragraphs which contain insufficient data than on paragraphs which contain superfluous or adequate data.

The mean gain of the Math. group (G) of 2.44 points is composed of mean gains of .68 on adequate data paragraphs, .48 on

superfluous data paragraphs, and 1.28 on missing data paragraphs.

The mean gain of the Non-Math. group (E) of 3.48 points is composed of mean gains of .88 on adequate data paragraphs, .92 on the superfluous data paragraphs, and 1.68 on the missing data paragraphs.

These differences are less significant than they appear because there was greater possibility of improvement on the insufficient data paragraphs, owing to lower initial scores of the pupils on them.

The conclusion seems justified that pupils of these groups did transfer, consciously or unconsciously, the attitudes and habits which they acquired in the training classes on special algebra problems, to non-mathematical problems in which similar abilities and habits were involved. Definitely cultivating with one group the ideas of selecting data from a mass, and of demanding data when missing, as general ideals, or some part of this training, gave the pupils an advantage which though small was real. The gain of the Non-Math. Group exceeded the gain of the Math. Group by 1.04 on the total scores.

Extent to Which Pupils Recognized Similarity Between Tests.—An attempt was made to ascertain the extent to which the pupils had recognized the similarity between the special problems of the algebra tests and those of the reading tests. At the close of the last test of the experiment, the pupils in those classes who had not been taught consciously to associate the mathematical and non-mathematical problems were asked to record any similarity which they observed between the two types of tests. Seventy replies were obtained from mathematics classes, and seventy from control classes.

Of 140 pupils replying, 22 per cent realized the similarity to the extent that they could express it clearly, 11 per cent claimed to see similarity but could not express it clearly; 22 per cent asserted that they saw similarity, but gave evidence that they did not realize the essential analogy; while 45 per cent asserted that they saw no similarity. The figures reveal that 67 per cent of these pupils did not express the similarity between the mathematical and the non-mathematical tests. A larger per cent of the control classes than of the mathematics classes claimed

to see similarity, although they failed to profit by it, *i.e.*, 38.6 per cent *versus* 27.1 per cent.

RECOGNITION BY PUPILS OF SIMILARITY BETWEEN TESTS

Classification of Replies	Per Cent in Mathe- matics Classes	Per Cent in Control Classes	Total Per Cent
1. Negative replies	64.3	25.7	45.0
2. Erroneous affirmative	8.6	35.7	22.0
3. Doubtful negative	2.8	18.6	11.0
4. Correct affirmative	24.3	20.0	22.0

Samples from these four classifications of replies follow:

SAMPLES OF REPLIES

1. Negative replies. (45%)
 - a. I see no connection.
 - b. No, these are common sense problems while the others are mathematical and can be answered by those only who have brains. Ye fortunates!
 - c. I see no connection unless it is to judge our understanding of the English language and our common sense.
2. Erroneous affirmative replies. (22%)
 - a. Both tests had facts in the paragraphs.
 - b. Both test our intelligence.
 - c. Hard thinking is required for both although more common sense is needed in this one (Reading test).
 - d. Yes, if we did not have an idea of mathematics we could not reason properly.
 - e. The only connection I see is that you must be careful in both tests.
3. Doubtful affirmative replies. (11%)
 - a. The reasoning is the same only the last test does not contain any numbers.
 - b. Both make you read clearly and catalogue your thoughts before answering.
 - c. You had to reason these problems out the same as the algebra problems had to be reasoned out.
 - d. I believe that if we are able to read these questions correctly we may be able to learn how to read algebra problems and understand them better.
4. Correct affirmative replies. (22%)
 - a. The only connection I see is to make you think and disregard such material as is useless, keeping before you the question in view.
 - b. There is a similarity because in these statements there is much

material that is not used in order to draw a conclusion of their meaning. So it was with the algebra problems.

- c. They both give a great many facts but one has to dig for the needed ones.
- d. The same idea is involved in answering questions with numbers or without. The paragraphs must be read thoroughly picking out the vital part which is the question asked for.
- e. Inability to do problems may have something to do with the fact that the problems are not read carefully. If there is a great many words that are unnecessary as in exercise 7, the pupils get all mixed up.
- f. Some contain too many facts, some too little.

Three Equivalent Groups of Forty-five Each.—The results of the scores of the Math., Non-Math., and Control Groups were further tested in three larger groups of forty-five pupils each, in which the composite scores of individual pupils were less closely matched. These groups are designated as Math. A, Non-Math. B, and Control C, and the original Math., Non-Math., and Control groups as group G, E, F. The range of difference between the scores of any one triplet of matched scores in no case amounts to 10 per cent of the total range of the composite scores for the groups; 68 per cent of the cases are within the limits of 3.2 per cent and 7.4 per cent of the total range. Composite scores were selected in such a manner as to give each group alternately the same advantage, and to produce groups which have like means and variabilities. After such matching, the scores of each group were listed in the order of size, regardless of the scores with which they were matched in the two other groups. This is the order in which they are reported subsequently.

The resulting groups, A, B, C, differ from Groups G, E, F, in three respects: (1) Twenty more pupils are included in each group, (2) the personnel of the first twenty-five is slightly altered, and (3) pupils with lower composite scores are included, as is indicated by the lower means of 503.8, 501.6 and 497.6, as opposed to 511.8, 511.24 and 511.12 of Groups G, E, and F, respectively.

The mean scores for these groups are uniformly lower than for Groups G, E, and F (see Tables XVII to XIX). The conclusions are approximately the same.

TABLE XVII

COMPOSITE SCORES OF THREE EQUIVALENT GROUPS, MATH. A, NON-MATH. B,
CONTROL GROUP C

A Math. Group	B Non-Math. Group	C Control Group	A Math. Group	B Non-Math. Group	C Control Group
677	670	668	492	501	514
660	658	646	491	493	512
641	624	643	489	491	508
611	617	609	487	489	503
602	601	602	478	485	500
588	579	583	475	482	489
587	561	570	474	476	487
579	551	555	473	460	478
572	551	555	472	458	452
562	544	555	461	451	444
559	538	553	441	451	435
558	537	549	430	446	435
553	536	546	428	441	427
552	533	545	425	439	427
547	531	542	422	409	427
531	531	540	417	406	424
529	524	537	416	399	410
509	522	537	410	396	410
504	519	534	400	396	373
498	515	527	376	358	371
495	513	521	374	348	353
494	512	520	336	345	334
494	508	519			
<i>M</i>			501.6	497.6	503.8
<i>SD</i>			76.7	76.1	76.4
<i>SD_M</i>			11.4	11.3	11.4

TABLE XVIII

DISTRIBUTION OF CHANGES IN INDIVIDUAL SCORES BETWEEN INITIAL AND FINAL TESTS AT WADLEIGH HIGH SCHOOL FOR MATH. A, NON-MATH. B, CONTROL GROUP C

A. MATHEMATICAL TESTS

Change	Total Problems (12)			Regular Problems (6)			Special Problems (6)		
	Groups			Groups			Groups		
	A	B	C	A	B	C	A	B	C
12	2	1	--	--	--	--	--	--	--
11	1	--	--	--	--	--	--	--	--
10	2	2	--	--	--	--	2	2	--
9	3	2	1	--	--	--	1	2	--
8	5	7	--	--	--	--	4	3	--
7	7	2	--	--	--	--	5	1	--
6	3	7	--	1	--	--	3	7	--
5	5	6	--	1	--	1	7	6	--
4	4	5	5	2	1	1	4	9	4
3	4	1	5	5	3	1	8	4	5
2	2	7	3	8	11	5	2	4	6
1	4	1	4	10	6	4	4	3	7
0	1	1	10	12	12	16	3	3	12
- 1	1	1	5	5	8	4	2	1	4
- 2	1	2	7	--	4	10	--	--	5
- 3	--	--	3	1	--	1	--	--	1
- 4	--	--	--	--	--	2	--	--	--
- 5	--	--	--	--	--	--	--	--	1
- 6	--	--	1	--	--	--	--	--	--
- 7	--	--	--	--	--	--	--	--	--
- 8	--	--	--	--	--	--	--	--	--
- 9	--	--	1	--	--	--	--	--	--
M	5.5	5.0	.24	1.2	.56	-.38	4.3	4.4	.62

THREE TYPES OF SPECIAL PROBLEMS

Change	Insufficient Data Problems (2)			Superfluous Infor- mation Problems (2)			Superfluous Number Problems (2)		
	Groups			Groups			Groups		
	A	B	C	A	B	C	A	B	C
4	4	8	--	4	3	1	12	4	2
3	2	--	--	3	6	1	4	7	1
2	23	22	6	10	12	5	12	14	10
1	1	2	2	8	7	--	6	9	5
0	14	12	30	9	10	26	10	8	22
- 1	--	1	3	9	6	3	1	2	3
- 2	1	--	4	1	1	9	--	1	2
- 3	--	--	--	1	--	--	--	--	--
M	1.4	1.7	.07	.87	1.2	-.09	1.9	1.6	.64

B. READING TESTS

Change	Total Problems (18)			Adequate Data (6)			Superfluous Data (6)			Insufficient Data (6)		
	Groups			Groups			Groups			Groups		
	A	B	C	A	B	C	A	B	C	A	B	C
8	1	1	--	--	--	--	--	--	--	--	--	--
7	1	5	--	--	--	--	--	--	--	--	--	--
6	1	1	--	--	--	--	--	--	--	--	--	--
5	1	5	--	--	1	--	--	1	--	1	1	--
4	8	9	2	--	1	--	--	--	--	2	3	--
3	5	10	4	2	3	1	2	3	2	3	5	1
2	11	5	7	5	12	2	7	10	1	11	14	2
1	7	4	14	16	9	14	15	10	18	8	13	15
0	7	4	9	16	15	18	14	12	11	15	8	18
- 1	2	1	2	6	3	8	6	9	12	3	1	4
- 2	1	--	4	--	1	1	1	--	1	2	--	5
- 3	--	--	2	--	--	1	--	--	--	--	--	--
- 4	--	--	1	--	--	--	--	--	--	--	--	--
M	2.2	3.4	.62	.58	1.0	.18	.60	.78	.27	1.0	1.6	.18

TABLE XIX

MEAN GAINS OF EQUIVALENT GROUPS, MATH. A, NON-MATH. B, CONTROL C

A. ALGEBRA TESTS

Group	Measure	Total Scores	Regular Prob- lems	Special Problems*			
				(a)	(b)	(c)	(d)
A Math.	M	5.49	1.16	4.33	1.44	.87	1.98
	SD	3.35	1.72	2.68	1.34	1.68	1.54
	SD _M	.50	.26	.38	.20	.25	.23
B Non-Math.	M	5.00	.56	4.42	1.71	1.18	1.56
	SD	3.18	1.51	2.58	1.39	1.52	1.37
	SD _M	.47	.23	.38	.21	.23	.20
C Control	M	.24	-.38	.62	.07	-.09	.64
	SD	2.97	1.94	2.01	1.00	1.36	1.32
	SD _M	.44	.29	.30	.15	.20	.20

B. READING TESTS

Group	Measure	Total Scores	Adequate Data	Superfluous Data	Missing Data
Math. G	<i>M</i>	2.18	.58	.60	1.00
	<i>SD</i>	2.06	.99	1.10	1.50
	<i>SD_M</i>	.31	.15	.16	.22
Non-Math. E	<i>M</i>	3.40	1.02	.78	1.60
	<i>SD</i>	2.18	1.38	1.36	1.27
	<i>SD_M</i>	.32	.21	.20	.19
Control F	<i>M</i>	.62	.18	.27	.18
	<i>SD</i>	1.79	1.06	1.08	1.10
	<i>SD_M</i>	.27	.16	--	--

* (a) = Total scores on special problems.

(b) = Scores on missing data problems.

(c) = Scores on superfluous information problems.

(d) = Scores on superfluous number problems.

TABLE XX

COMPARISONS OF MEAN GAINS AMONG GROUPS A, B, C OF ALGEBRA and READING TESTS

A. ALGEBRA TESTS

Group	Measure	Total Scores	Regular Prob.	Special Problems*			
				(a)	(b)	(c)	(d)
A vs. C Math. vs. Control	<i>M</i> (A)	5.49	1.16	4.33	1.44	.87	1.98
	<i>M</i> (C)	.24	-.38	.62	.07	-.09	.64
	<i>D</i>	5.25	1.54	3.71	1.37	.96	1.34
	<i>SD_D</i>	.67	.39	.48	.19	.32	.31
	<i>EC</i>	2.84	1.43	2.74	2.58	1.08	1.58
B vs. C Non-Math. vs. Control	<i>M</i> (B)	5.00	.56	4.42	1.71	1.18	1.56
	<i>M</i> (C)	.24	-.38	.62	.07	-.09	.64
	<i>D</i>	4.76	.94	3.80	1.64	1.27	.92
	<i>SD_D</i>	.64	.37	.48	.26	.31	.28
	<i>EC</i>	2.66	.91	2.81	2.31	1.49	1.16
B vs. A Non-Math. vs. Math.	<i>M</i> (B)	5.00	.56	4.42	1.71	1.18	1.56
	<i>M</i> (A)	5.49	1.16	4.33	1.44	.87	1.98
	<i>D</i>	-.49	-.60	.09	.27	.31	-.42
	<i>SD_D</i>	.69	.35	.54	.24	.35	.31
	<i>EC</i>	-.26	-.63	.06	.40	.33	-.49

B. READING TESTS

Group	Measure	Total Scores	Adequate Data	Superfluous Data	Missing Data
A vs. C Math. vs. Control	<i>M</i> (A)	2.18	.58	.60	1.00
	<i>M</i> (C)	.62	.18	.27	.18
	<i>D</i>	1.56	.40	.33	.82
	<i>SD_D</i>	.41	.22	.23	.27
	<i>EC</i>	1.37	.66	.52	1.09
B vs. C Non-Math. vs. Control	<i>M</i> (B)	3.40	1.02	.88	1.60
	<i>M</i> (C)	.62	.18	.27	.18
	<i>D</i>	2.78	.84	.61	1.42
	<i>SD_D</i>	.42	.26	.26	.25
	<i>EC</i>	2.40	1.17	.86	2.06
B vs. A Non-Math. vs. Math.	<i>M</i> (B)	3.40	1.02	.88	1.60
	<i>M</i> (A)	2.27	.58	.60	1.00
	<i>D</i>	1.13	.44	.28	.60
	<i>SD_D</i>	.45	.26	.26	.29
	<i>EC</i>	.91	.61	.39	.74

(a) = Total scores
(b) = Missing data

(c) = Superfluous information.
(d) = Superfluous number.

That the inclusion of the larger number of forty-five pupils in Groups A, B, and C, did not alter markedly the conclusions which were drawn from the results of the tests with smaller groups of pupils in Groups G, E, and F, is indicated in the foregoing tables and the following paragraphs.

In the algebra tests both the Math. Group and the Non-Math. Group showed positive gains in the total scores of 5.49 and 5.00, respectively. The Control Group showed a slight but positive gain of .24. These gains may be compared with the gains of 6.24, 5.92, and .60 of the smaller groups. The net gains of the larger Math. and Non-Math. groups were 5.25 and 4.76 as opposed to the net gains of the smaller groups of 5.64 and 5.32.

In addition to the analysis which was made of the data for Groups G, E, and F, further analysis was made of the algebra test scores of Groups A, B, and C, to ascertain how the total scores on the special problems were affected by each of the different types of special problems. The total scores on the six special problems are composed of scores on two problems which contain superfluous numbers, two which contain superfluous information, and two which contain insufficient data. Analysis of the gains shows the following division:

The average gain of Math. Group (A) of 4.33 points on special

problems is composed of gains of 1.44 on problems with missing data, .87 on problems with superfluous information, and 1.98 on problems with superfluous numbers.

The average gain of the Non-Math. Group (B) of 4.42 points on special problems is composed of gains of 1.71 on problems with missing data, 1.18 on problems with superfluous information, and 1.56 on problems with superfluous numbers.

The average gain of the Control Group (C) of .62 points on the special problems is composed of .07 on the problems with missing data, —.09 on problems with superfluous information, and .64 on problems with superfluous numbers.

As the relative difficulty of different types of special problems is not known, no comparison is attempted of the relative influence of the different types. It is evident that pupils in the groups which received instruction made gains in all types of special problems and that the Control Group did not make marked gains in any one type of special problems.

In the reading tests Groups A and B showed positive gains, slightly lower but practically equivalent to those made by Groups G and E. Compare the gains of 2.42 and 3.48 for G and E, with 2.18 and 3.40 for A and B, respectively. The larger Control Group C made a positive gain of .62 as opposed to .52 by the smaller Control Group F.

So far as these studies permit, we may generalize by saying that pupils can be taught to select from the data of an algebra problem those which are essential for its solution and to demand sufficient data when they are missing from problems with which they are familiar. A small amount of instruction is required to cause pupils to exercise choice if the instruction is definite. Exasperation and confusion are less apparent when pupils have been told of the character of the problems and have been warned to expect them. On the contrary, the pupils of these groups appeared to enjoy the diversion which the problems furnished, and to take pride in their ability to solve them.

The selection of data is, of course, not algebra; neither is it history nor science, but it belongs to a method of solving problems wherever they occur. Having learned to expect superfluous and insufficient data in algebra problems and to select the data for their solution, these pupils improved in ability to select data for problems in other fields of learning which involved similar

specific habits. The pupils whose attention had been drawn to the analogy between problem solving in different fields, were more successful in selecting data for problems in other fields than the one in which they had been specifically trained.

It should be emphasized that the non-mathematical problems were similar to the mathematical problems in form of statement and in the manner of their presentation. They differed primarily in the presence or absence of mathematical content. In the words of one of the pupils, "The same idea is involved in answering questions with or without numbers. The paragraphs must be read thoroughly, picking out the vital part which is the question asked for."

A striking illustration of the transfer is furnished by non-mathematical problem 8, relative to a proposed bridge over the Hudson River or three tunnels under it. In the initial test no pupil out of 195 pupils gave a reply which indicated a feeling of need for more information. Those who answered the question, unhesitatingly chose one plan or the other and gave arguments to support it. In the final test eight pupils out of the Non-Math. Group, and none out of the other groups, gave such answers as: "I know nothing about tunnels or bridges so cannot say which is best," and "Of course, I think a bridge is safer, but I cannot reach a conclusion as not enough facts are given."

If a pupil can be taught consciously to scrutinize and make selection from the data given in problems, regardless of the fields in which they occur, he may avoid waste of time and discouragement in discovering for himself more or less thoroughly, a method which he probably will use when confronted by problems of life which require discrimination among the data. While the results of this phase of this study are sufficiently reliable to be significant, more extended studies would afford criticism of the conclusions and determine the extent to which applications are possible.

CHAPTER V

SUMMARY AND PRACTICAL CONCLUSIONS

In so far as these studies represent general conditions, they indicate: (1) that it is not a common practice to require first-year high school pupils to exercise discrimination in the choice of data for the solution of algebra problems, (2) that it is possible, by means of specific instruction, so to teach pupils that they consciously exercise this desirable function, (3) that when pupils have been taught to select data necessary for the solution of mathematical problems, they improve in ability to select data for problems in other fields of learning.

Text books in algebra give few problems which contain more or less data than are needed to solve them, and teachers generally do not use such problems. The introduction of problems containing superfluous or insufficient data is disconcerting to pupils who have been accustomed to problems containing only the necessary data. Comparatively few pupils readily adjust themselves to the novel problems and successfully complete their solution. This is especially true when pupils are wholly unaware of the character of the special problems. When their character is explained and pupils are put on guard for the occurrence of such problems, much of their difficulty is removed. A small amount of drill is needed to teach pupils to choose the necessary data from among superfluous data or to detect the absence of data when they are lacking from problems with which pupils are already familiar.

The practical conclusions of this study rest upon certain principles of modern psychology and education. That training acquired in the school room may be utilized in life outside of school is a basic assumption of formal education. The school is an artificial situation in which pupils are presented with problems and are caused to have experiences, under conditions which are less wasteful than those of natural learning through incidental experiences of life. The effectiveness of edu-

cation in schools depends largely upon the degree to which school room conditions and procedure can be made identical with those outside of school.

It is a truism that the predominance in life of problematic situations and the ever-changing character of social demands make it an important function of the school to cultivate the individual's powers of reflective thinking. It is recognized, however, that the power to think is not a general function to be developed by indiscriminate mental gymnastics. It consists rather of specific mental processes and habits which may be defined and developed by suitable instruction. These habits become general through widespread experience from acquisition of specific habits in each of many fields. Since it is practically certain that the school cannot improve the native equipment of pupils, the work of the teacher is to enable pupils who do think to think well by employing proper methods and by forming useful habits.

Few persons would attempt to justify the algebra now taught wholly by claims of its direct practical applications, but claims have long been made of its cultural and disciplinary values. Until recently the validity of such disciplinary values have been questioned by psychologists because of uncertainty concerning the possible or probable transfer of training. Psychological experiments of recent years have established the possibility of transfer from one field of learning to another of specific identical elements. The effectiveness of this transfer depends largely upon the methods of teaching and learning. These statements are generally accepted as proven:

- (1.) Change in one function alters any other function only in so far as the two have identical elements. Change in the second function is merely the result of the alteration of those of its elements which were elements also of the first function, and so were changed by its training.
- (2.) This identity may be of all degrees of generality and of several different types. There may be identity of content, of method of procedure, or it may be of still more general character and be in terms of attitudes or ideals.
- (3.) The presence of identical elements is not in itself a warrant of transfer. It may take place, but it need not do so. Transfer is most sure to occur in cases of identity of sub-

stance and least likely in cases of attitudes and ideals. The likelihood of transfer is increased when the learner is conscious of the connections and when practice occurs in several types of situations.

Traditional instruction in algebra has been based on faith in the value of mathematical computations for computations' sake. The belief that if computations are mastered all other phases of problem solving will take care of themselves is shown by such statements as, "Our main problem is not that of teaching pupils what data are to be selected, but what to do with them when proper data are given. If the pupils understand this, they will experience little or no difficulty in selecting the proper data in a real life problem." Faith in the superior value of abstract problems and in the automatic transfer of ability to think, developed in connection with abstract problems, may be the explanation of the tendency among teachers to omit verbal problems and to emphasize computations. The teacher who said, "There are so many fundamental skills which must be developed that we have not time for problem material," must have felt that "fundamental skills" have other functions than to serve as means for solving verbal problems.

Such faith and practices are not in harmony with experimentally determined laws of learning and transfer of training. If teachers of algebra, or any subject, are to accomplish training in thinking through the instrumentality of that subject, it is necessary that specific mental processes and connections be singled out for specific exercise. The mental operations which comprise reasoning are essentially the same in the many varieties of problems that are encountered whether in practical affairs or in school subjects of study. Among the habits involved in successful problems solving are: those of getting a problem clearly in mind, suspending judgment, scrutinizing the available data, detecting the absence of necessary data when they are missing, choosing those data which are pertinent from redundant items.

Accordingly, it seems desirable to include among the problems which are presented to pupils, enough problems which require selection and discovery of data that pupils become accustomed to this important phase of solving genuine problems.

The task of improving the abilities involved in problem solving is much like the task of improving other abilities, such as skill in the fundamental operations of addition, multiplication, and factoring, or in typewriting, or in reading a foreign language. There must be opportunity for abundant practice in solving many problems of different types. We learn to think by thinking, but if a learner is left to his own devices he is likely to improve slowly and he may fail to develop many elements of effective technique. Superior results are obtained when pupils are taught consciously to apply processes and are given practice under guidance.

In view of these considerations, problems containing superfluous or insufficient data should be used only after pupils have been definitely told of their nature, and have received instruction and frequent practice in the technique of solving them. After such instruction pupils may be prepared to solve problems of the special types when they occur unannounced among other problems.

The presence of problems which contain superfluous and insufficient data need not conflict with the need for systematic drill by means of problems which contain precisely the data needed to solve them. The necessity for drill has been amply demonstrated in scientific investigations. The consensus of opinion among teachers and writers of texts is that for purposes of drill, the regular problems should be kept separate from the special problems here under consideration. It may be desirable to eliminate from problems all elements except those which should be considered, when the aim of instruction is to perfect skill in fundamental operations. Problems which require selection of data may be introduced after pupils are familiar with the more simple situations.

SUMMARY OF CONCLUSIONS

An important function of education in school is to teach pupils to think better, by training them in the specific habits involved in problem solving.

It is desirable that instruction, the aim of which is to teach pupils to do reflective thinking, should include training in ability to detect the absence of essential data when they are lacking,

and to select from a mass of data those which are pertinent to the solution of the problem.

The verbal problems of algebra furnish suitable medium for this training which has not been fully utilized.

It is important to make problem solving in school as nearly like problem solving out of school as the purposes of classroom instruction at the time permit.

Single problems which contain superfluous or insufficient data represent many problematic situations in life to such a degree that we are justified in organizing and presenting to pupils in school many such problems as an additional means of training in the choice of data.

The value of algebra problems for teaching pupils to think depends largely upon the methods of instruction. Pupils learn to think usefully by learning better methods of thinking.

When a pupil has acquired a habit of analyzing problems and of selecting the necessary data in algebra problems, he tends to treat similar problems in other studies in the same way.

Methods of thinking which are consciously acquired and used are more likely to be carried over from one field of study to another and to life than would be less definite methods of which pupils may be only partially aware.

Problems which contain superfluous or insufficient data should be the subject of special exercises the purpose of which is to make pupils aware of the nature of the special problems, and to give practice in their solution. Pupils may then be expected to succeed when such problems occur incidentally among problems to which they have been accustomed.

APPENDIX

I

ALGEBRA TESTS

School.....ClassDate
Write your name here.....
When did you begin to study algebra?.....

Directions: Look at the practice problem below. Read it carefully. Then in the space below it write out the solution of the problem. Do all your work on this paper. Write the answer on the dotted line.

PRACTICE PROBLEM

A company of 95 soldiers is to be divided into three squads, of which the second contains 5 more men than the first, and the third contains 10 more men than the second. How many men will there be in each squad?

Answer.....

On the following pages are several problems similar to this one. In the space below each problem, work out the problem. Do all your work on this paper.

Work as rapidly as possible until you have finished, or until you are told to stop. Do not spend so much time on one problem that you do not have time to try them all. If you cannot solve a problem go on to the next one.

You may write what you think about a problem on this paper, but *do not ask any questions aloud*. You must decide for yourself what to do. If for any reason you do not answer a problem, tell why you do not.

N. B.—In the following samples of test forms, space for solution of problems is omitted.

PROBLEMS OF FORM ALPHA

PART I

1. A piece of iron 52 inches long is to be cut into two parts so that one part is 4 inches longer than the other. What will be the length of each part?
2. A stenographer spends one-third of her monthly income for room and board, one-sixth for clothes, and one-fourth for other expenses. She saves \$30. What is her income?

3. A bus can carry 10 more passengers than an auto. To accommodate all the passengers arriving on a certain train 7 autos and 2 buses will be needed. How many passengers are arriving?
4. Water flowing through an inlet pipe can fill a tank in 30 minutes. It can be emptied by an outlet pipe in 40 minutes. If the tank is empty and both pipes are opened, how long will it take to fill the tank?
5. A certain recipe for fruit punch calls for twice as many oranges as lemons, and proportional amounts of sugar, grape juice and water. If oranges are 35 cents a dozen and lemons are 25 cents a dozen, how many dozen of each must be used in making a punch at a total cost of \$4.75?
6. The members of a history class of fifty pupils have agreed each to buy a supplementary book. A book company is advertising a new book which has been adopted by many schools and which is well liked. The selling price is \$2. It is printed on good paper, is well bound and contains 300 pages of reading matter. The principal can secure the books for the class for $\frac{2}{3}$ of the selling price, if he buys a large number of them. How much should each pupil pay for his book?

PART II

7. One number exceeds another number by 16. If 3 times the first number be added to 5 times the second number the result is 85. What are the two numbers?
8. A father earns twice as much in a day as his son. The father worked 6 days and the son worked 5 days, for which they received \$34. How much should each receive?
9. A clerk spends one-fourth of his yearly salary for room and board, one-eighth for clothes, and one-sixth for other expenses. He saves \$880 which he invests in bonds bearing $4\frac{1}{2}$ per cent interest. What is his yearly salary?
10. A furniture dealer wishes to make a profit of 40 per cent of the selling price on a shipment of dining-room sets for which he paid \$156 each. At what price should he sell them in order to realize this profit?
11. The capacity of a certain swimming pool is 133,000 gallons. It can be filled by one pipe which has 5 outlets in 40 hours, and by a second pipe which has only 3 outlets, in 60 hours. It is desired to fill the pool as quickly as possible. How long should it require if both pipes are running at the same time?
12. The first appropriation for a library for Congress was made in 1800. The present Library of Congress was completed in 1897 at a total cost of six million dollars, the gold leaf on the dome alone costing \$3800. If the first appropriation had been \$1200 less than it was it would have paid only for the gold leaf on the present library. What must have been the amount of the first appropriation?

PROBLEMS OF FORM BETA

PART I

1. It is desired to divide a class of 51 pupils into two groups, one of which shall contain 3 more than the other. How many pupils will each group contain?
2. Mr. Moore decided to divide a sum of money among three investments. He invested one-third of it in oil stock, one-half in bank stock, and the remaining \$1000 in Liberty Bonds. How much did he invest?
3. In order to accommodate all of the pupils who went on a sleigh-ride party last winter, 5 large sleighs and 3 small sleighs were needed. Each large sleigh carried twice as many persons as a small sleigh. How many pupils were there in the party?
4. A certain job of printing can be done on one printing press in 50 minutes and on another in 40 minutes. How long will it take if the job is put on both presses at the same time?
5. A certain recipe for fruit punch calls for twice as many oranges as lemons, and proportional amounts of sugar, grape juice and water. If oranges are 35 cents a dozen and lemons are 25 cents a dozen, how many dozen of each must be used in making a punch at a total cost of \$4.75?
6. The girls of the basket ball squad, consisting of ten members, have decided to buy new uniforms for themselves. The Spalding Company advertises a uniform which the girls all like, and which sells for \$10. The material is of very good quality, and the uniform is well made. The colors are blue and gold. The athletic director can obtain the lot of ten uniforms for $\frac{4}{5}$ of the selling price if cash is paid. How much will each girl have to pay?

PART II

7. One number exceeds a second number by 5. If 4 times the first number be added to 3 times the second number the result is 160. What are the two numbers?
8. \$75 were distributed among 3 boys and 4 girls. If each girl received 3 times as much as each boy, how much did each receive?
9. A man has invested a sum of money in three enterprises. One-half of it is invested in bonds paying 4 per cent interest, one-third in bonds paying 3 per cent, and the remaining \$2500 in bonds paying 5 per cent. How much has he invested?
10. A dealer wishes to make a profit of 35 per cent on the selling price of a tube radio set for which he paid \$60. What should he charge for the set in order to realize this profit?
11. A certain manuscript contains 30 pages. A typist who can write at the rate of 60 words a minutes, can copy this manuscript in 4 hours.

Her assistant, whose rate is 40 words a minute, can do it in 6 hours
How long would it take the two of them?

12. The first high school for girls in New York City was organized in 1897. At the present time there are three large high schools for girls enrolling more than thirteen thousand girls. The senior class at Wadleigh High School alone totals 250 girls. If the number of pupils in the first high school had been 100 less than it was, it would have been only two times as great as the present senior class at Wadleigh. What must have been the number of pupils in the first high school for girls?

II

READING TESTS*

Name.....School.....
 Class.....Date.....

Directions: Read each of the following paragraphs carefully, and answer the question below it if you can. When possible the answers are to be found in the paragraphs. You may read the paragraphs as many times as you need to, but do not spend so much time on one that you do not have time for the others.

1. Nearly fifteen thousand of the city's workers joined in the parade of September seventh, and passed before two hundred thousand cheering spectators. There were workers of both sexes but the men far outnumbered the women.

What did the people who looked at the parade do when it passed by?

2. The Northmen probably came to America as early as the year 1000, nearly 500 years before Columbus and the Cabots sailed from Europe. There is no record of any one else having come to America before the year 1000.

By whom do you think America was first discovered?.....

3. Both before and after Christmas, Bob Adams worked harder than he did in the spring, summer, or fall. Only rarely did he reach home before eleven o'clock; and on every morning except Sunday he was up at six, dressed and done with breakfast by a quarter of seven, left the house at ten minutes of seven, and reached Mr. Clark's store at ten minutes of eight. In spite of the long hours and hard work, he was happy because his pay was raised twice.

What did Bob do between six and seven A.M. six days of the week?

3. A new kind of fruit-jar is advertised. A woman decides to give it a trial and buys one; but the fruit she cans in it spoils.

Was she right in concluding that the new kind of fruit-jar is not a success? Tell why.

*The following key indicates the type of problem in each of the paragraphs of the Reading Tests.

Adequate Data Paragraphs	- - - - -	2	5	11	13	15	18
Superfluous Data Paragraphs	- - - - -	1	3	7	9	10	17
Insufficient Data Paragraphs	- - - - -	4	6	9	12	14	16

5. It may seem at first thought that every boy and girl who goes to school ought to do all the work that the teacher wishes done. But sometimes other duties prevent even the best boy or girl from doing so. If a boy's or girl's father died and he had to work afternoons and evenings to earn money to help his mother, such might be the case. A good girl might let her lessons go undone in order to help her mother by taking care of the baby.
What does this paragraph prove?
.....

6. The *Evening News* gives an account of some trouble in a room on Seventh Avenue. It seems that two men entered a room where several persons were sitting at a table, and in the scuffle that followed they killed one man and severely injured two others.
What punishment should these men receive?
.....

7. Hay-fever is a very painful, though not a dangerous disease. It is like a very severe cold in the head, except that it lasts much longer. The nose runs; the eyes are sore; the person sneezes; he feels unable to think or work. Sometimes he has great difficulty in breathing. Hay-fever is not caused by hay, but by the pollen from certain weeds and flowers. Only a small number of people get this disease, perhaps one in fifty. Most of those who do get it, can avoid it by going to live in certain places during the summer and fall. Almost every one can find some place where he does not suffer from hay-fever.
During what seasons of the year would a person have the disease described in the paragraph?
.....

8. New York City is very much in need of better facilities for getting people across the Hudson River into New Jersey. It is estimated that a bridge across the river would cost ten million dollars. For the same amount of money three tunnels could be constructed under the river.
Which way should the money be spent? Why?
.....

9. For nearly thirty years Lewis Carroll was a lecturer on mathematics at Oxford. He studied divinity and occasionally preached, but his shy and retiring nature, together with a tendency to stammer, kept him from the regular ministry. He gave many lectures to audiences made up mainly of children. These lectures were of various sorts, but consisted principally of narratives from his books, illustrated by lantern pictures. He invented a number of mathematical games.
What sort of lectures did Lewis Carroll give when his audiences were not made up mainly of children?
.....

10. According to the *Kansas City Star*, the farmers of Kansas are too prosperous to trouble themselves about careful harvesting and threshing. They do not cut the fields clean. A gleaner 80 years old, after

the wheat harvest in Pawnee County last year, went over the wheat fields with a wagon, a rake, a brush, and a shovel, and gathered up nine hundred bushels in forty days, and sold it at a dollar a bushel. Do you think that most of the wheat which the farmers grew was left on the ground by the harvesters? Give reasons for your answer.

.....

11. During the winter of 1608-10 in the Jamestown Colony, rats, mice, and snakes were eaten by the colonists and relished, and fungi were eaten. It is even reported that an Indian who had been killed in an assault on the stockade was eaten by the poorer men. From this what do you conclude must have happened to the Colony that winter?
-

12. A scientist observed that the Bulgarians were an unusually long-lived people. In trying to discover the reason for the fact, he searched for something in their living conditions different from the conditions of other people. He found that the Bulgarians drank much more buttermilk than other people.

Was this scientist right in concluding that the drinking of buttermilk was responsible for the long lives of the Bulgarians; and that if other races took up the custom of drinking as much buttermilk as the Bulgarians, they would live equally as long lives?

Explain your answer.

13. During the Revolutionary War, France helped the Amercian colonies with both men and ships. Ten years after the close of this war, France was again at war with England.

What would the French people think the United States ought to do?

.....

14. The story is told that during the 17th Century in Europe, a man bade his wife farewell, one day, saying, "Goodbye, I am going off to the Thirty Years War."

Why could this not be true?

15. During the year of 1824, 8000 immigrants came to America. During the year 1844, 78,000 immigrants came. During the year 1854, 427,000 immigrants came.

What do these statements show about immigration?

.....

16. In a certain city plans were about completed for the building of a City Hospital. Several prominent citizens unexpectedly began to oppose the plan, with the result that it had to be given up.

Do you think that these citizens should be blamed for opposing the building of a hospital? Why?

.....

17. In the spring men begin to work the soil. Those who have small gardens break up the ground with such tools as spades and forks. Those who live on farms turn the soil over with plows drawn by

horses. In these ways the soil is loosened. After the planting is done, the weeds must be killed and the soil must be loosened. In places where little rain falls, other ways of watering the plants must be found. When must the soil be loosened a second time?.....

18. At the close of the Civil War, many of the Southern negroes would not return to work on the plantations for pay, but wanted land of their own. There was also a scarcity of white laborers in the South, and but little capital with which to buy agricultural machinery. What effect would you expect these conditions to have on the size of the farms in the South?
.....

PLAN FOR SCORING READING TEST

Answers were called correct or incorrect according to the following standards. In general the answer must show that the pupil has read the paragraph and the question, and is able to word a response to both.

(a) = Samples of correct answers.

(b) = Samples of incorrect answers.

1. A response to the essential element of the question, and a correct conclusion in light of the data of the paragraph.
 - a. They cheered.
 - b. They joined in the parade.
 They went on their way.
 They wondered where all the ladies are.
 Thought of the good work they had done for our city.
 Watched to see if there was anyone they knew.
 Clapped their hands and waved their handkerchiefs.
 Took off their hats when the flag went by, if there was one.
2. Use of all of the data to reach a consistent conclusion.
 - a. The Northmen.
 Columbus, because it says the Northmen probably came.
 - b. Columbus, or the Cabots.
 History teaches that Columbus did and I still believe it.
3. Selection of those data only which are pertinent to question.
 - a. Dressed, ate breakfast, left the house.
 - b. (Anything more or less than (a) is incorrect).
4. Indication of a realization of the need for more data.
 - a. No, one trial is not sufficient, she should have tried another one.
 She may not have followed directions.
 She may not have prepared the fruit properly.
 - b. A categorical reply:
 No, the fruit was spoiled before she put it in.
 She should have given the lid another turn.
 Yes, because the fruit she canned in it spoiled.

5. A reasonable conclusion which involves consideration of all the data in the paragraph.
 - a. There are cases where a pupil need not do all the work a teacher wishes done.
 - b. They were good children.
A boy may work after school.
It isn't always your own fault if you are not bright.
6. A demand for more information, or indication that the pupil realizes the need for it.
 - a. It depends upon the reasons for the killing and who the men were.
It seems that the men should be put to death, but one must know the causes before punishing.
 - b. The electric chair or life imprisonment.
The punishment for second degree murder.
They should be handled firmly but kindly so they will not hate the law.
7. Selection from a mass of data of a few pertinent data.
 - a. Summer and fall.
 - b. (Both summer and fall are necessary.)
When the flowers bloom.
8. Request for more information or hesitation about deciding.
 - a. I know nothing about tunnels or bridges, so cannot say which is best.
I do not know which is the best one, but the most useful and convenient one should be used.
Of course I think a bridge is safer, but I cannot reach a conclusion as not enough facts are given.
 - b. The money should be spent for tunnels. They are cheaper and the money saved could be contributed toward something the city needs almost as much.
More people could be accommodated by tunnels, traveling in three tunnels would take less time than on one bridge.
A bridge would be healthier and safer, and a bridge is a thing of beauty.
9. Selection of data pertinent to the question.
 - a. Mathematics, or Mathematics and divinity.
Mathematics but rather preachy.
 - b. Uninteresting and about mathematics.
His audiences were mainly children.
Narratives and pictures.
10. Response to a fundamental element in the paragraph and formulation of a conclusion (1) which is consistent with this element, (2) can be justified by the data, (3) or by general principles which pupils know from previous study.
 - a. (1) Responses to "prosperous."
No, if so they would not be so prosperous.

Prosperous men would not be so careless as to leave more than they harvest.

- (2) Responses to "amount gathered in a given time."

No, it took one man such a long time to gather what he did.

- (3) Responses "amount gathered vs. total raised."

No, 900 bushels is comparatively a small amount.

No, they have enormous fields and harvest great amounts.

Harvesting machines are more efficient than that.

The farmers would protest.

- b. Yes, because they do not cut the fields clean.

Yes, it says they were too prosperous.

An old man gathered so much.

11. Use of all the data to reach a consistent conclusion without introducing extraneous considerations.

- a. There must have been a famine.

They were starving or were very hungry.

They must have died from the effects of the food eaten.

- b. People were too lazy to work and ate what came along.

A very hard and bitter winter.

12. Realization that more evidence is needed to prove casual relationship.

- a. There may have been other less obvious causes.

If he would look farther into the matter, I think he would find that something else makes them live longer, if they do.

You cannot say Yes or No. There are not enough facts from which you may draw a conclusion.

- b. (Categorical reply with a specific solution is incorrect.)

13. Use of all the data to reach a consistent conclusion.

- a. France would expect aid in return.

- b. Not if they were reasonable.

14. Indications that pupil realizes the man did not have sufficient data for making such a remark.

- a. How could he know how long the war would last?

- b. It is a story, and all stories are not true.

No war could last 30 years.

A man could not fight 30 years and live.

There was no such war.

It did not occur in the 17th century.

15. Use of all the data to reach a consistent reply. Disregard of subtle suggestions.

- a. Immigration was increasing.

Increasing as the "years-go-by."

- b. Immigration increased each year.

Increased every 20 years, or day by day.

16. A demand for more data, or realization of need for them.

- a. They may have had good reasons why it was a better thing to do.

Maybe Yes, maybe No, you have to understand the situation.

- b. Yes, a hospital is a benefit to the poor and should be supported.
It shows what wealth can do.
17. Selection from a mass of data of the essential element.
Disregard of subtly connected elements.
 - a. After planting is done.
 - b. After planting and the weeds are killed.
In the spring.
To keep the weeds from growing.
18. Use of all the data to reach a consistent conclusion.
 - a. The farms would be smaller.
Farms would be smaller but more numerous.
 - b. Farmers would not be so progressive.
There were less farms.

III

OUTLINE OF SPECIAL INSTRUCTION IN SOLVING PROBLEMS CONTAINING SUPERFLUOUS AND INSUFFICIENT DATA *

LESSON I

PROBLEMS WHICH LACK ESSENTIAL DATA

- A. The pupils in an algebra class tried to solve this problem. They could not do it? Could you?

A freight train left Kansas City for St. Louis running at the rate of 12 miles an hour. At the same time a passenger train running at the rate of 45 miles an hour left St. Louis for Kansas City. How long will it be before the trains meet?

What additional facts are needed? They learned that the distance between the two cities is 285 miles, then they found that the trains would meet in 5 hours.

- B. Many of the problems which we have to solve in life are like this problem. In order to solve them it is necessary to have additional facts. If the additional facts cannot be found the problem cannot be solved. In the following problems what facts are needed?

1. A woman took $15\frac{1}{2}$ dozen eggs to a grocery store. She received 10 cents a dozen for them. How many pounds of sugar could she get for the eggs?
2. A Ford can make 5 more miles on a gallon of gasoline than a Dodge car. At 25 cents a gallon what is the cost of enough gasoline to run a Ford a mile?
3. The main waiting room in the Union Station at Washington, D. C., has an area of 28,600 square feet. The length exceeds the width by 90 feet. How many people can be accommodated in this waiting room?
4. Charles, John and Henry are brothers; Charles' age is 2 years greater than John's age. Three years from now Henry will be half as old as Charles. How old are the boys now?
5. Out of every ten thousand men in an army 920 will probably be exposed to typhoid fever; 350 out of every thousand exposed to fever will probably fall sick. In an army of 150,000 men, how many probably will die of typhoid?

- C. In the future you may expect to find problems of this sort. Do not try to solve a problem until you are sure you have all the necessary facts.

* Suggestions for the form of instruction were obtained from: Schorling and Clark, *Mathematics for the Eighth Year*.

LESSON II

PROBLEMS WHICH CONTAIN MORE FACTS THAN ARE
NEEDED

- A. Many pupils can solve problems when the necessary facts are given and no others, and when they know exactly what rule or principle to use. They are lost if they have to think about the problem and decide for themselves which facts to use and how to use them.

Out-of-school problems do not come to us exactly organized with only the necessary facts present. Often the problem contains numbers and information which are not needed for the solution. In such cases we have to separate the problem from the unessential facts. We do this by first getting the problem clearly in mind, then selecting the facts which are important for the solution.

- B. The following problems will give practice in this kind of thinking and problem solving.
1. A boy made 50 toy boats. He sold 24 of them at 30 cents each, 10 at 25 cents each, 6 at 20 cents each, and the rest at 15 cents each. How much did he receive for those he sold at 30 cents each?
 2. Forty-five boys were hired to pick apples from 15 trees in an apple orchard. In fifty minutes each boy had picked 48 choice apples. If all the apples picked were packed away carefully in 8 boxes of equal size, how many apples were put in each box?
 3. Mrs. Jones spent 13 hours making a dress. The material cost \$7, the trimmings, buttons and thread costing \$1.25 more than the dress-goods. If Mrs. Jones' time is worth 40 cents an hour, what did the work on the dress cost?
 4. The elevation of Mt. Whitney in California, the highest recorded point in the United States, is 14,501 feet measured from the sea level. The lowest point of dry land in the United States is Death Valley which also is in California. If 52 times the elevation of Death Valley be decreased by 45 and the result be increased by the elevation of Mt. Whitney, the sum is zero. What is the elevation of Death Valley?
 5. A speculator bought a farm of 360 acres on the edge of a town for \$72,000. After keeping it for two months, he sold it to a development company to be cut up into small farms and city lots. The company paid him \$90,000 for the whole farm. How much did he gain on the deal?

LESSON III

ANALYSIS OF PROBLEMS *

- A. Recall the types of problems which we have been studying.

In order to be better prepared to meet problems of either kind, those which contain more facts than are needed and those in which essential facts are missing, practice in analyzing problems is needed.

- B. Read each of the following problems carefully then answer the questions below it. Four answers are given to each question. Put a check mark in front of the best answer to each question.

1. Mrs. Smith bought a vacuum cleaner for \$70 to be paid in monthly installments. Two months she failed to make the payments but each time she made it up with the next payment. How many months was it before the cleaner was paid for if her payments were \$5 per month?

Which of the following facts are you asked to find out?

- a. The amount of each payment
- b. The number of payments missed
- c. How long it took to pay for the cleaner
- d. The final cost of the cleaner

Which of the following facts are given in the problem?

- a. The depreciation of the cleaner
- b. The cost of the repairs
- c. The cost of the cleaner
- d. The number of the payments

Which of these facts are needed for solution of the problem?

2. Mr. Anderson who is a lawyer has an annual income of \$4000. He employs in his office a bookkeeper, a stenographer, and an office boy. He can afford to spend only \$40 a week for office help. How much can he afford to pay each of them? He must pay the office boy \$5 a week, and he estimates that the stenographer's work is worth two-thirds as much as the bookkeeper's.

Which of the following facts are you asked to find out?

- a. How much money Mr. Anderson makes a week
- b. How much more the stenographer earns than the bookkeeper
- c. How much he pays each of his help a week
- d. The amount of money spent in a year

Which of the following facts are given in the problem?

- a. The city in which the lawyer lived
- b. The ratio of the salaries of his office helpers
- c. The amount Mr. Anderson makes in a year
- d. The total amount paid per week to office help

Which of these facts are needed in solution of the problem?

3. The City Hall of Philadelphia is said to cover a greater area than any other building in the United States. One-fifth of its width

* Adapted from P. R. Stevenson, *Problem Analysis Tests*.

exceeds one-sixth of its length by 13 feet; and one-ninth of its length exceeds one-tenth of its width by 7 feet. How does this compare with the size of Madison Square Garden in New York City?

Which of the following facts are you asked to find out?

- a. The dimensions of the City Hall
- b. The size of Madison Square Garden
- c. The cost of the two buildings
- d. The difference in the areas covered by the buildings.

Which facts are given in the problem?

- a. The area covered by the City Hall?
- b. The length and width of Madison Square Garden
- c. When these two buildings were constructed
- d. The relations between the length and width of City Hall

What facts are needed for the solution of the problem?

Are enough facts given?

4. The Valley Dale Company bought 180 acres of land adjacent to the city for \$54,000. They divided it into 200 lots of equal value which they sold at \$500 each. What did each of the 200 lots cost the company?

Which of the following things are you asked to find out?

- a. The cost of each lot
- b. The number of acres in the land purchased
- c. The selling price of each lot
- d. The profit made by the company

Which of the following facts are given in the problem?

- a. The cost of each lot
- b. The selling price of the whole tract
- c. The number of lots
- d. The profit made by the company

Which facts are needed for the solution of the problem?

5. Plants feed upon certain plant foods present in the soil, such as potash, nitrogen, and phosphoric acid. A fair crop of potatoes will remove from an acre of ground about 99 pounds of these three foods. How many pounds of each do potatoes remove if the amount of potash is 5 times that of the phosphoric acid and the amount of nitrogen is $2\frac{1}{4}$ times that of the phosphoric acid?
 - a. What are you asked to find out in this problem?
 - b. What plant foods do potatoes feed upon?
 - c. What does the problem tell about how much more nitrogen is needed than phosphoric acid?
 - d. How much plant food does a fair crop of potatoes take from an acre of ground?
 - e. How much is a fair crop of potatoes?

LESSON IV

PRACTICE PROBLEMS

A. *Some things worth remembering.* Our experience has shown us the following things about problems:

1. Problems may contain unnecessary facts in the form of extra numbers which need not be used.
2. Problems may contain information about the problem which need not be used in the solution.
3. Some problems do not contain enough facts to solve the problem, and no answer can be found.
4. Other problems do not contain enough facts, but the answer can be represented algebraically by an algebraic expression.
5. Most problems in textbooks contain no extra statements and all of the necessary facts, and can be solved.
6. If we first carefully read and analyze a problem we can tell what kind of a problem it is.

B. *To which of the above classes do the following problems belong?*

1. If a certain number is diminished by 8 and the remainder is multiplied by 6, the product equals 126. Find the number.
2. The four highest buildings in New York City are the Municipal, the Singer, the Metropolitan Life, and the Woolworth buildings. The Singer is 52 feet higher than the Municipal, the Metropolitan is 88 feet higher than the Singer, and the Woolworth is 92 feet higher than the Metropolitan. Their total height is 2664 feet. How high is each?
3. A certain even number increased by 7 is equal to 63. What is the number?
4. If six times the area of Lake Superior, the largest fresh water lake in the world, be decreased by 12,000 square miles the result equals the area of the Caspian Sea, the largest salt water lake in the world, which covers 180,000 square miles. What is the area of Lake Superior?
5. A man had plans for a house whose foundation was 12 feet longer than it was wide. Finding that it would cost \$18,000, which was more than he could afford, he decided to cut off 3 feet from the length and 2 feet from the width. In this way he lowered the cost to \$15,000. What were the dimensions of the foundation before and after making the changes?
6. I buy some bulbs at a seed store for \$3, paying 75 cents per dozen for one variety and 50 cents per dozen for another variety. The number of the first variety exceeds the number of the second variety by 18. How much change should I receive from a five-dollar bill?
7. The length of a tennis court is 42 feet more than the width, and the perimeter is 228 feet. What are the dimensions of the tennis court?

8. A vegetable bed is 4 feet longer than it is wide. One-half of it is planted in beans, one-half of the remainder in radishes, and the rest in lettuce. What is the size of each of the three sections?
9. Forty-eight children from a certain school paid 10 cents apiece to ride 7 miles on the cars to the woods. There in a few hours they gathered 2765 nuts. 605 of these were bad but the rest were shared equally among the children. How many good nuts did each one get?
10. Chicago and Madison are 140 miles apart. Suppose a train starts from each city toward the other at the same time, one at the rate of 35 miles an hour and the other at the rate of 40 miles an hour. How soon will they meet?
11. Two automobiles leave Pittsburgh at the same time, one traveling east, and the other traveling west at a rate which exceeds that of the east-bound auto by 5 miles an hour. How far apart will they be in 3 hours?
12. A man travels from one city to another, a distance of 171 miles, in 6 hours. If he returns at the same rate how long will it take him to return one-third of the distance?
13. The quotient obtained when a certain number is decreased by 4 and the remainder divided by 4, equals the quotient obtained when the sum number is increased by 4 and the sum divided by 6. Find the number.
14. From Cincinnati, Ohio, to Cleveland by way of Columbus is 262 miles. A train leaving Cleveland at 4:50 will be in Columbus at 7:50 and in Cincinnati at 11:00. If the distance from Cleveland to Columbus is 140 miles, how far is it from Cincinnati to Columbus?
15. When a six-foot pole casts a shadow $4\frac{1}{2}$ feet long, how long is the shadow of an eight-foot pole?
16. A flag pole which is 13 inches in diameter at the base tapers to a diameter of 6 inches at the top. The shadow cast by the pole is 80 feet. At the same time the shadow of a man who is 6 feet tall and is standing 30 feet from the pole is 8 feet long. How high is the flag pole?

IV

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